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## EVALUATION OF QUALITY INDICATORS OF FUNCTIONING CYBER PROTECTION MANAGEMENT SYSTEMS OF INFORMATION SYSTEMS

*Evidence of the complexity of the cybersecurity problem is the rapid increase in the number of information security breaches and losses on cybersecurity threats combined with an increase in the average loss from each of the breaches. Therefore, it is necessary to create requirements for a cybersecurity system that could provide more opportunities in the choice of methods in the management of the protection of automated information systems.*

*The task of determining the optimal quality indicators of information resource management systems of automated systems is one of the most important problems in designing integrated information security systems. This is due to the complexity of such systems, the presence of many variable parameters, and the complexity of calculating quality indicators. In addition, the determined quality indicators should not only ensure the optimality of the target function, but also the stability of the protection system in a wide range of external adverse effects. The problem is that the existing methods of calculating integrated quadratic estimates (IQE) do not take into account errors in determining quality indicators, as well as the vector nature of these indicators.*

*The aim of this work is to solve problems (development of algorithms), which are a problem of optimization of stable protection management systems using vector objective functions. Based on the model of information management system protection of information resources in the form of an automatic control system, the method of forming integrated quadratic estimates (IQE) of control error is proposed. This method takes into account the weights of the estimates at the desired installation time and standard transfer functions. Algorithms for calculating IQE according to the modified Katz formula and Ostrom's method for arbitrary order control systems are developed, including vector representation of the objective function of the protection system. The vector penalty function is proposed and the algorithm of its calculation is developed to display the degree of infringement of conditions of stability of parameters of the system of protection by the Rauss-Hurwitz criterion.*

*Key words: integrated quadratic estimates (IQE) of control error, arbitrary order control systems, vector objective functions, the algorithm for calculating the vector penalty function, and integrated information security systems.*

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## ОЦІНКА ПОКАЗНИКІВ ЯКОСТІ ФУНКЦІОНУВАННЯ СИСТЕМ УПРАВЛІННЯ КІБЕРЗАХИСТОМ ІНФОРМАЦІЙНИХ СИСТЕМ

*Підтвердженням складності проблеми кібербезпеки є досить швидке зростання кількості порушень інформаційної безпеки та витрат на загрози кібербезпеки, у поєднанні з ростом середнього збитку від кожного із порушень. Тому необхідно сформулювати вимоги до системи кіберзахисту, яка б могла надавати більше можливостей у виборі методів в управлінні захистом автоматизованих інформаційних систем.*

*Завдання визначення оптимальних показників якості систем управління захистом інформаційних ресурсів автоматизованих систем є однією з найважливіших проблем проектування комплексних систем захисту інформації. Це зумовлено складністю подібних систем, наявністю безлічі варіюваних параметрів, складністю обчислення показників якості. Крім того, визначувані показники якості повинні не лише забезпечувати оптимальність цільової функції, але і стійкість функціонування системи захисту в широкому діапазоні зовнішніх несприятливих впливів. Проблема полягає в тому, що існуючі методи обчислення ІКО не враховують помилки визначення показників якості, а також векторний характер цих показників.*

*Метою даної роботи є рішення завдань (розробка алгоритмів), що становлять проблему оптимізації стійких систем управління захистом при використанні векторних цільових функцій. На основі моделі уявлення системи управління захистом інформаційних ресурсів у вигляді системи автоматичного управління, запропоновано спосіб формування інтегральних квадратичних оцінок (ІКО) помилок управління, що враховує вагові коефіцієнти оцінок за бажаним часом встановлення і стандартні передавальні функції. Крім того, розроблені алгоритми обчислення ІКО за модифікованою формулою Каца і методом Острьома для систем управління довільного порядку, включаючи векторне уявлення цільової функції системи захисту. Запропонована векторна штрафна функція і розроблений алгоритм її обчислення для відображення ступеня порушення умов стійкості параметрів системи захисту за критерієм Рауса-Гурвиця.*

*Ключові слова: інтегральні квадратичні оцінки (ІКО) помилок управління, системи управління довільного порядку, векторні цільові функції, алгоритм розрахунку векторної штрафної функції, інтегровані системи захисту інформації.*

### Introduction

Creating and ensuring the functioning of an effective cybersecurity system is a complex and multifaceted process that requires considerable effort. At the same time, cybersecurity provided by the state should not slow down the process of forming a national information space that would correspond to the information and intellectual potential of the state and would not hinder Ukraine's entry into the world information space as a subject of equal international relations. In view of this, the strategic task of state policy on cybersecurity should be to form a system based on scientifically sound criteria and experience in information security management.

Modern information systems are complex and therefore dangerous in themselves, even without the active actions of attackers. New vulnerabilities in software and hardware implementations are constantly being identified. We have to take into account the extremely wide range of hardware and software, the many connections between components and more.

Evidence of the complexity of the cybersecurity problem is the parallel (and fairly rapid) increase in the number of information security breaches and losses on cybersecurity threats combined with an increase in the average loss from each of the breaches. The latter circumstance is another argument in favor of the importance of cybersecurity.

Therefore, it is necessary to create requirements for a cybersecurity system that could provide more opportunities in the choice of methods in the management of the protection of automated information systems.

It should be noted that in Ukraine a single information system is being created, which consists of permanent elements:

- central subsystem;
- functional subsystems;
- transport data network;
- data processing centers, telecommunication networks of system subjects;
- complex information protection systems of subsystems of a single system.

Therefore, this system is distributed. It should be noted that this complicates the task very much. And since in addition to a single system there are other (regional, local, departmental and other) systems, this issue becomes very important and difficult in the system of government, in the banking system and other spheres of society [1, 2].

In addition, in modern conditions the large number of cyberattacks on systems of all classes should be taken into account.

The task of determining the optimal quality indicators of information resource management systems of automated systems is one of the most important problems in designing integrated information security systems. This is due to the complexity of such systems, the presence of many variable parameters, and the complexity of calculating quality indicators. In addition, the determined quality indicators should not only ensure the optimality of the target function, but also the stability of the protection system in a wide range of external adverse effects.

In [3,4] the models of both systems themselves and their control systems and information protection systems are given. But it should be noted that these systems provide very serious mistakes and errors in countering modern cyberattacks.

In the paper [5] it is proposed to use the provisions of the theory of automatic control systems as a model of the protection management system, in which the indicators are given in the form of integrated quadratic estimates (IQE) of control error [6; 7]. The problem is that the existing methods of calculating IQE [8-12] do not take into account errors in determining quality indicators, as well as the vector nature of these indicators.

The most suitable model that was used to solve this problem is the model given in [13]. This model allows to solve all problems which stand at management of cybersecurity of information systems in modern difficult conditions.

The aim of the work is to solve problems (development of algorithms), which are a problem of optimization of stable protection management systems using vector objective functions.

### Main part

IQE management error has the form

$$I = \int_0^{\infty} e^2(t) dt. \tag{1}$$

Control error  $e(t)$  can be defined as the weighted sum of the derivatives of the deviation  $z(t) = y(\infty) - y(t)$  of the controlled value  $y(t)$  from the set value  $y(\infty) = 1$  using weights  $\omega_k, k = \overline{0, l}; \omega_0 = 1$ :

$$e(t) = \sum_{k=0}^l \omega_k \cdot z^{(l-k)}(t). \quad (2)$$

Let the control function  $y(t)$  correspond to the transfer function

$$W(s) = \frac{\beta(s)}{\alpha(s)}; \alpha(s) = \sum_{i=0}^n \alpha_i s^{n-i}; \beta(s) = \sum_{j=0}^m \beta_j s^{m-j}; \alpha_n = \beta_m. \quad (3)$$

The Laplace transform  $z_k(s)$  for derivatives  $z^{(k)}(t)$  in expression (2) is obtained by applying the theorem on the differentiation of the original with the initial conditions  $z(0) = 1; z^{(k)}(0) = 0, k = \overline{1, l}$ :

$$z_0(s) = \frac{1-W(s)}{s}; z_k(s) = -s^{k-l} \cdot W(s), k = \overline{1, l}; l \leq n - m.$$

Converting Laplace equation (2) and introducing a polynomial with weights

$$\omega(s) = \sum_{k=0}^l \omega_k \cdot s^{l-k}, \quad (4)$$

get the error image:

$$E(s) = \frac{1-W(s) \cdot \omega(s)}{s}. \quad (5)$$

Considering expressions (3) for the transfer function and determining through

$$\delta(s) = \frac{\alpha(s) - \beta(s) \cdot \omega(s)}{s}, \quad (6)$$

the error image is represented as the ratio of two polynomials:

$$E(s) = \frac{\delta(s)}{\alpha(s)}. \quad (7)$$

In the general case, the error can be defined as the difference between the reference function  $y_e(t)$  and controlled variable  $y(t)$ :  $e(t) = y_e(t) - y(t)$ .

The transfer function of the reference function is set as  $W_e(s) = 1/\omega(s)$ , then the error image will look like

$$E(s) = \frac{W_e(s) - W(s)}{s}. \quad (8)$$

After transformations we come to equality  $E(s) = \delta(s)/\alpha(s)/\omega(s)$ . To determine the reference function, choose a standard transition function  $y_0(t)$  of the desired form with known values of the setting time  $\tau_0$  and coefficients of the transfer function  $W_0(s) = 1/\gamma(s)$  [4; 5]:

$$\gamma(s) = \sum_{k=0}^i \gamma_k \cdot s^{i-k}, \gamma_0 = 1.$$

Setting the value of the time of setting the reference function  $\tau_e$ , calculate the weights  $\omega_k = \mu^{l-k} \cdot \gamma_k, k = \overline{0, l}$ , where  $\mu = \tau_e/\tau_0$ . According to these formulas and expressions (6), (7) we will make an algorithm for forming an error image.

**Algorithm 1.** Input parameters:  $\tau_e$  – the time of setting the reference function,  $\tau_0$  – the standard process setting time,  $\gamma$  – an array of coefficients of the standard transition function,  $\alpha$  and  $\beta$  – arrays of coefficients of the denominator and numerator of the transfer function. Output parameters:  $\delta$  – an array of error image numerator coefficients.

Step 1. Put  $l := \dim \gamma - 1; n := \dim \alpha - 1; \mu := \tau_e / \tau_0; \eta := 1; k := l$ .

Step 2. Calculate  $\omega_k := \gamma_k \cdot \eta; \eta := \eta \cdot \mu$ .

Step 3. If  $k > 0$ , put  $k := k - 1$  and go to step 2.

Step 4. Calculate the convolution of two vectors  $c := \beta \otimes \omega$ .

Step 5. Calculate  $\delta := \alpha - c$ .

Step 6. The end.

To calculate the IQE (1) based on error conversion  $E(s)$  by Parseval's theorem we write

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} E(s) \cdot E(-s) ds. \tag{9}$$

Let us represent the error image by equality

$$E(s) = \frac{\delta(s)}{\alpha(s)}, \tag{10}$$

where  $\alpha(s)$  – polynomial of degree  $n$ , all the roots of which lie in the left half-plane,  $\beta(s)$  – polynomial of degree  $n-1$ . Then

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{\delta(s) \cdot \delta(-s)}{\alpha(s) \cdot \alpha(-s)} ds. \tag{11}$$

Denote by  $\delta(s^2) = \beta(s) \cdot \beta(-s)$  polynomial of even degrees and rewrite (11):

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{\delta(s^2)}{\alpha(s) \cdot \alpha(-s)} ds. \tag{12}$$

Let in the image (10)

$$\alpha(s) = \sum_{i=0}^n \alpha_i s^i; \beta(s) = \sum_{i=0}^{n-1} \beta_i s^i. \tag{13}$$

Considering  $m = n - 1$ , we present the polynomial of even degrees (12) in the form

$$\delta(s^2) = \sum_{k=0}^m \delta_k s^{2k}.$$

Integral (12) is calculated by the modified formula of A.M. Katz [5]:

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{\delta(s^2)}{\alpha(s) \cdot \alpha(-s)} ds = \frac{(-1)^{n-1}}{2\alpha_n} \cdot \frac{G}{H}, \tag{14}$$

where  $H$  – Hurwitz determinant for a polynomial  $\alpha(s)$ , and  $G$  – determinant obtained by  $H$  replacing

the last line with a line of polynomial coefficients  $\delta(s^2)$ :

$$H = \begin{vmatrix} \alpha_0 & \alpha_2 & \alpha_4 & \alpha_6 & \dots & 0 \\ 0 & \alpha_1 & \alpha_3 & \alpha_5 & \dots & 0 \\ 0 & \alpha_0 & \alpha_2 & \alpha_4 & \dots & 0 \\ 0 & 0 & \alpha_1 & \alpha_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & \alpha_{n-1} \end{vmatrix}; G = \begin{vmatrix} \alpha_0 & \alpha_2 & \alpha_4 & \alpha_6 & \dots & 0 \\ 0 & \alpha_1 & \alpha_3 & \alpha_5 & \dots & 0 \\ 0 & \alpha_0 & \alpha_2 & \alpha_4 & \dots & 0 \\ 0 & 0 & \alpha_1 & \alpha_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \delta_0 & \delta_1 & \delta_2 & \delta_3 & \dots & \delta_{n-1} \end{vmatrix} \tag{15}$$

Formula (14) differs from Katz's formula by writing polynomials (13) in ascending order of the degree of the variable  $s$ , which simplifies the calculation algorithm IQE. Considering

$\alpha_i^{(0)} = \alpha_i, i = \overline{0:2:n}; \alpha_i^{(1)} = \alpha_i, i = \overline{1:2:n}$  (here is the record of the species  $i = \overline{k:2:n}$  means a sequence of integers beginning with  $k$ , increases with step 2 and does not exceed  $n$ ), convert the determinants (15) to a triangular shape:

$$\lambda^{(1)} = \frac{\delta_0}{\alpha_0}; \delta_j^{(1)} = \delta_j - \lambda^{(1)} \cdot \alpha_i, (i, j) = (2, 1), (4, 2), (6, 3), \dots \quad (16)$$

$$\gamma^{(k)} = \frac{\alpha_{k-2}^{(k-2)}}{\alpha_{k-1}^{(k-1)}}; \lambda^k = \frac{\delta_{k-1}^{(k-1)}}{\alpha_{k-1}^{(k-1)}}, k = \overline{2, n-1}; \quad (17)$$

$$\alpha_i^{(k)} = \alpha_i^{(k-2)} - \gamma^{(k)} \cdot \alpha_{i+1}^{(k-1)}; \delta_j^{(k)} = \delta_j^{(k-1)} - \lambda^{(k)} \cdot \alpha_{i+1}^{(k-1)}, \quad (i, j) = (k, k), (k+2, k+1), (k+4, k+2), \dots \quad (18)$$

In this case IQE (15) is calculated by the formula

$$I = \frac{(-1)^{n-1} \cdot \delta_{n-1}^{(n-1)}}{2 \cdot \alpha_n \cdot \alpha_{n-1}^{(n-1)}}. \quad (19)$$

The algorithm for calculating the IQE according to Katz's formula looks like this.

**Algorithm 2.** Input parameters:  $\alpha$  and  $\beta$  – arrays of denominator and numerator coefficients of error conversion. Output parameter:  $I$  – IQE.

Step 1. Put  $n := \dim \alpha - 1; m := n - 1; \delta := PolPow2(\beta)$  ( $PolPow2$  – a function that calculates an array of coefficients of a polynomial of even degrees  $\delta(s^2)$ ).

Step 2. Calculate  $\lambda := \delta_0 / \alpha_0$  and put  $j := 1; i := 2$ .

Step 3. Calculate  $\delta_j := \delta_j - \lambda \cdot \alpha_i$  and put  $j := j + 1$ .

Step 4. If  $i < m$ , then put  $i := i + 2$  and go to step 3.

Step 5. Put  $s := 1; k := 2$ .

Step 6. Calculate  $\gamma := \alpha_{k-2} / \alpha_{k-1}; \lambda := \delta_{k-1} / \alpha_{k-1}$ , and put  $s := -s; j := k; i := k$ .

Step 7. Calculate  $\alpha_i := \alpha_i - \gamma \cdot \alpha_{i+1}; \delta_j := \delta_j - \lambda \cdot \alpha_{i+1}$  and put  $j := j + 1$ .

Step 8. If  $i < m$ , put  $i := i + 2$  and go to step 7.

Step 9. If  $k < m$ , put  $k := k + 1$  and go to step 6.

Step 10. Calculate  $I := \delta_{n-1} / (2 \cdot \alpha_{n-1} \cdot \alpha_n)$ .

Step 11. If  $s < 0$ , then set  $I := I$ .

Step 12. The end.

Integral (11) can be calculated by the method of Ostrom using an iterative procedure [11]. Let the polynomials in the image of the error (10) have the form

$$\alpha(s) = \sum_{i=0}^n \alpha_i s^{n-i}; \beta(s) = \sum_{i=0}^{n-1} \beta_i s^{n-i}. \quad (20)$$

Considering

$$\alpha_i^{(0)} = \alpha_i, i = \overline{0:2:n}; \alpha_i^{(1)} = \alpha_i, i = \overline{1:2:n}; \beta_i^0 = \beta_i, i = \overline{0:2:n-1};$$

$$\beta_i^{(1)} = \beta_i, i = \overline{1:2:n-1},$$

calculate  $\gamma^{(k)}$  and  $\lambda^{(k)}$  for  $k = \overline{1, n}$ :

$$\gamma^{(k-1)} = \frac{\alpha_{k-2}^{(k-2)}}{\alpha_{k-1}^{(k-1)}}; \lambda^{(k-1)} = \frac{\beta_{k-2}^{(k-2)}}{\alpha_{k-1}^{(k-1)}}, k = \overline{2, n-1}; \quad (21)$$

$$\alpha_i^{(k)} = \alpha_i^{(k-2)} - \gamma^{(k-1)} \cdot \alpha_{i+1}^{(k-1)}; \beta_i^{(k)} = \beta_i^{(k-2)} - \lambda^{(k-1)} \cdot \alpha_{i+1}^{(k-1)}, i = \overline{k:2:n-1}; \quad (22)$$

$$\gamma^{(k)} = \frac{\alpha_{k-1}^{(k-1)}}{\alpha_k^{(k)}}; \lambda^{(k)} = \frac{\beta_{k-1}^{(k-1)}}{\alpha_k^{(k)}}, k = \overline{n-1, n}. \quad (23)$$

The value of IQE (11) is calculated by the formula

$$I = 0,5 \cdot \sum_{k=1}^n \frac{(\lambda^{(k)})^2}{\gamma^{(k)}}. \quad (24)$$

**Algorithm 3.** Input and output parameters are similar to algorithm 2.

Step 1. Put  $n := \dim \alpha - 1; m := n - 1; I := 0; k := 2$ .

Step 2. Calculate  $\gamma := \alpha_{k-2} / \alpha_{k-1}; \lambda := \beta_{k-2} / \alpha_{k-1}; I := I + \lambda^2 / \gamma$  and put  $i := k$ .

Step 3. Calculate  $\alpha_i := \alpha_i - \gamma \cdot \alpha_{i+1}; \beta_i := \beta_i - \lambda \cdot \alpha_{i+1}$ .

Step 4. If  $i < m$ , put  $i := i + 2$  and go to step 3.

Step 5. If  $k < m$ , then put  $k := k + 1$  and go to step 2.

Step 6. Put  $k := m$ .

Step 7. Calculate  $\gamma := \alpha_{k-1} / \alpha_k; \lambda := \beta_{k-1} / \alpha_k; I := I + \lambda^2 / \gamma$ .

Step 8. If  $k < n$ , put  $k := k + 1$  and go to step 7.

Step 9. Calculate  $I = 0,5 \cdot I$ .

Step 10. The end.

The input parameters of algorithms 2 and 3 are calculated by algorithm 1.

A necessary condition for the stability of the protection management system is to require the positivity of all coefficients  $\alpha_i > 0, i = \overline{0, n}$  [2; 3].

Considering  $\alpha_i^{(0)} = \alpha_i, i = \overline{0:2:n}; \alpha_i^{(1)} = \alpha_i, i = \overline{1:2:n}$ , the Rauss-Hurwitz stability criterion is reduced to calculations by formulas

$$\gamma^{(k)} = \frac{\alpha_{k-2}^{(k-2)}}{\alpha_{k-1}^{(k-1)}}; \alpha_i^k = \alpha_i^{k-2} - \gamma^{(k)} \cdot \alpha_{i+1}^{(k-1)}, i = \overline{k:2:n-1}; k = \overline{2, n-1}. \quad (25)$$

We introduce the notation of the elements of the Raus-Hurwitz determinant:

$$\rho_0 = \alpha_0, \rho_1 = \alpha_1, \rho_k = \alpha_k^k, k = \overline{2, n-1}, \rho_n = \alpha_n. \quad (26)$$

**Algorithm 4.** Input parameter:  $\alpha$  – an array of coefficients of the characteristic polynomial. Output parameter:  $\rho$  – an array of Raus-Hurwitz coefficients.

Step 1. Put  $n := \dim \alpha - 1; m := n - 1; \rho := \alpha; k := 2$ .

Step 2. Calculate  $\gamma := \rho_{k-2} / \rho_{k-1}$  and put  $i := k$ .

Step 3. Calculate  $\rho_i := \rho_i - \gamma \cdot \rho_{i+1}$ .

Step 4. If  $i < m$ , put  $i := i + 2$  and go to step 3.

Step 5. If  $k < m$ , put  $k := k + 1$  and go to step 2.

Step 6. The end.

Necessary and sufficient conditions for the stability of the protection management system are the conditions  $\alpha_i > 0, i = \overline{0, n}$ , and  $\rho_k > 0, k = \overline{2, n-1}$ . Taking into account these conditions and algorithm 4, we will develop an algorithm for determining the stability of the control system according to the Rauss-Hurwitz criterion.

**Algorithm 5.** Input parameter:  $\alpha$  – an array of coefficients of the characteristic polynomial. Output parameter:  $B$  – a sign of system stability.

Step 1. Put  $n := \dim \alpha - 1; B := 1; i := 0$ .

- Step 2. If  $\alpha_i \leq 0$ , put  $B := 0$  and go to step 10.  
 Step 3. If  $i < n$ , put  $i := i + 1$  and go to step 2.  
 Step 4. Put  $m := n - 1$ ;  $\rho := \alpha$ ;  $k := 2$ .  
 Step 5. Calculate  $\gamma := \rho_{k-2} / \rho_{k-1}$  and put  $i := k$ .  
 Step 6. Calculate  $\rho_i := \rho_i - \gamma \cdot \rho_{i+1}$ .  
 Step 7. If  $i < m$ , put  $i := i + 2$  and go to step 6.  
 Step 8. If  $\rho_k \leq 0$ , put  $B := 0$  and go to step 10.  
 Step 9. If  $k < m$ , put  $k := k + 1$  and go to step 5.  
 Step 10. The end.

Let the coefficients of the characteristic polynomial-function of the vector of the varied parameters of the system  $\alpha_i = \alpha_i(x)$ ,  $i = \overline{0, n}$ ;  $x \in R^p$ . In order for the system to be stable, it is necessary and sufficient to:

$$\alpha_i(x) > 0, i = \overline{0, n}; \rho_k(x) > 0, k = \overline{2, n-1}. \quad (27)$$

The degree of violation of the first group of inequalities is represented by one penalty function

$$P(x) = \sum_{i=0}^n \max \{-\alpha_i(x), 0\}. \quad (28)$$

Inequalities (27) correspond to the areas of constraints:

$$\Omega_1 = \{x | \alpha_i(x) > 0, i = \overline{0, n}\}; \Omega_k = \{x | \rho_k(x) > 0\}, k = \overline{2, n-1}. \quad (29)$$

Let's make a system of areas from them:  $D_1 = \Omega_1$ ,  $D_k = D_{k-1} \cap \Omega_k$ ,  $k = \overline{2, n-1}$ , from which by means of differences of sets we will adjust areas of levels of restrictions:

$$H_0 = R^p \setminus D_1; H_k = D_k \setminus D_{k+1}, k = \overline{1, n-1}; H_{n-1} = D_{n-1}. \quad (30)$$

A two-dimensional vector penalty function is proposed for the transition to the stability region:

$$F(x) = \begin{cases} (0; P(x)), & x \in H_0; \\ (k; -\rho_{k+1}(x)), & x \in H_k, k = \overline{1, n-2}; \\ (n-1; 0), & x \in H_{n-1}. \end{cases} \quad (31)$$

The first component reflects the affiliation of the argument to a particular area and is called a *level function*. The second is the penalty for violating the restriction and is called a *penalty function*. When all the constraints are satisfied (the third component), the level function acquires the maximum value  $n-1$  and the penalty function is reset.

**Algorithm 6.** Input parameter:  $\alpha$  – an array of coefficients of the characteristic polynomial. Output parameter:  $B$  – a sign of system stability;  $F$  – the value of the vector penalty function.

- Step 1. Put  $n := \dim \alpha - 1$ ;  $B := 1$ ;  $i := 0$ ;  $h := 0$ ;  $P := 0$ .  
 Step 2. If  $\alpha_i \leq 0$ , calculate  $P := P - \alpha_i$  and put  $B := 0$ .  
 Step 3. If  $i < n$ , put  $i := i + 1$  and go to step 2.  
 Step 4. If  $B = 0$ , put  $F := (h, P)$  and go to step 12.  
 Step 5. Put  $m := n - 1$ ;  $\rho := \alpha$ ;  $k := 2$ .  
 Step 6. Calculate  $\gamma := \rho_{k-2} / \rho_{k-1}$  and put  $h := h + 1$ ,  $i := k$ .  
 Step 7. Calculate  $\rho_i := \rho_i - \gamma \cdot \rho_{i+1}$ .  
 Step 8. If  $i < m$ , put  $i := i + 2$  and go to step 7.  
 Step 9. If  $\rho_k \leq 0$ , put  $F := (h, -\rho_k)$ ;  $B := 0$  and go to step 12.

Step 10. If  $k < m$ , put  $k := k + 1$  and go to step 6.

Step 11. Calculate  $F := (h : 1, 0)$ .

Step 12. The end.

In the case of estimating the vector objective function, we assume that the transfer function (3) depends on the vector of variable parameters  $X$ :

$$W(x, s) = \frac{\beta(x, s)}{\alpha(x, s)}, \alpha(x, s) = \sum_{i=0}^n \alpha_i(x) s^{n-i}; \quad (32)$$

$$\beta(x, s) = \sum_{j=0}^m \beta_j(x) s^{m-j}; \alpha_n(x) = \beta_m(x).$$

Defining the weight polynomial (4) and determining the polynomial (6)

$$\delta(x, s) = \frac{\alpha(x, s) - \beta(x, s) \cdot \omega(s)}{s}, \quad (33)$$

IQE is presented as a function of variable parameters:

$$I(x) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{\delta(x, s) \cdot \delta(x, -s)}{\alpha(x, s) \cdot \alpha(x, -s)} ds. \quad (34)$$

At each value of the vector  $X$  from the stability of the system values of this function can be calculated by algorithms 2 and 3. Redefine the vector penalty function (31) into a vector objective function, supplementing it with the value of IQE (34) in the field of stability:

$$F(x) = \begin{cases} (0; P(x)), & x \in H_0; \\ (k; -\rho_{k+1}(x)), & x \in H_k, k = \overline{1, n-2}; \\ (n-1; I(x)), & x \in H_{n-1}. \end{cases} \quad (35)$$

Modify algorithms 2 and 3 according to algorithm 6 to calculate the vector objective function (35) [10].

**Algorithm 7.** Input parameter:  $\alpha$  and  $\beta$  – arrays of denominator and numerator coefficients of error conversion. Output parameter:  $F$  – the value of the vector objective function;  $B$  – a sign of system stability.

Step 1. Put  $n := \dim \alpha - 1; B := 1; i := 0; h := 0; P := 0$ .

Step 2. If  $\alpha_i \leq 0$ , calculate  $P := P - \alpha_i$  and put  $B := 0$ .

Step 3. If  $i < n$ , put  $i := i + 1$  and go to step 2.

Step 4. If  $B = 0$ , put  $F := (h, P)$  and go to step 17.

Step 5. Calculate  $\delta := \text{PolPow2}(\beta); \lambda := \delta_0 / \alpha_0$  and put  $m := n - 1; j := 1; i := 2$ .

Step 6. Calculate  $\delta_j := \delta_j - \lambda \cdot \alpha_i$  and put  $j := j + 1$ .

Step 7. If  $i < m$ , put  $i := i + 2$  and go to step 6.

Step 8. Put  $s := 1; k := 2$ .

Step 9. Calculate  $\gamma := \alpha_{k-2} / \alpha_{k-1}; \lambda := \delta_{k-1} / \alpha_{k-1}$  and put  $h := h + 1; s := -k; j := k; i := k$ .

Step 10. Calculate  $\alpha_i := \alpha_i - \gamma \cdot \alpha_{i+1}; \delta_j := \delta_j - \lambda \cdot \alpha_{i+1}$  and put  $j := j + 1$ .

Step 11. If  $i < m$ , put  $i := i + 2$  and go to step 10.

Step 12. If  $\alpha_k \leq 0$ , put  $F := (h, -\alpha_k); B := 0$  and go to step 17.

Step 13. If  $k < m$ , put  $k := k + 1$  and go to step 9.

Step 14. Calculate  $I := \delta_{n-1} / (2 \cdot \alpha_{n-1} \cdot \alpha_n)$ .

Step 15. If  $s < 0$ , put  $I := -I$ .

Step 16. Calculate  $F := (h + 1, I)$ .

Step 17. The end.



Algorithm for calculating the vector objective function according to Ostrom.

**Algorithm 8.** Input and output parameters are similar to the algorithm 7.

Step 1. Put  $n := \dim \alpha - 1$ ;  $B := 1$ ;  $i := 0$ ;  $h := 0$ ;  $P := 0$ .

Step 2. If  $\alpha_i \leq 0$ , calculate  $P := P - \alpha_i$  and put  $B := 0$ .

Step 3. If  $i < n$ , put  $i := i + 1$  and go to step 2.

Step 4. If  $B = 0$ , put  $F := (h, P)$  and go to step 15.

Step 5. Put  $m := n - 1$ ;  $I = 0$ ;  $k := 2$ .

Step 6. Calculate  $\gamma := \alpha_{k-2} / \alpha_{k-1}$ ;  $\lambda := \beta_{k-2} / \alpha_{k-1}$ ;  $I := I + \lambda^2 / \gamma$  and put  $h := h + 1$ ;  $i := k$ .

Step 7. Calculate  $\alpha_i := \alpha_i - \gamma \cdot \alpha_{i+1}$ ;  $\beta_i := \beta_i - \lambda \cdot \alpha_{i+1}$ .

Step 8. If  $i < m$ , put  $i := i + 2$  and go to step 7.

Step 9. If  $\alpha_k \leq 0$ , put  $F := (h, -\alpha_k)$ ;  $B := 0$  and go to step 15.

Step 10. If  $k < m$ , put  $k := k + 1$  and go to step 6.

Step 11. Put  $k := m$ .

Step 12. Calculate  $\gamma := \alpha_{k-1} / \alpha_k$ ;  $\lambda := \beta_{k-1} / \alpha_k$ ;  $I := I + \lambda^2 / \gamma$ .

Step 13. If  $k < n$ , put  $k := k + 1$  and go to step 12.

Step 14. Calculate  $F := (h + 1, 0, 5 \cdot I)$ .

Step 15. The end.

Studies have shown the high efficiency of the proposed algorithms. And on the night of February 23-24, 2022, they effectively worked against DDoS attacks carried out by Russian hackers on Ukrainian information systems.

### Conclusions

Based on the model of information management system protection of information resources in the form of automatic control system:

1. The method of forming integrated quadratic estimates (IQE) of control error is proposed. This method takes into account the weights of the estimates at the desired installation time and standard transfer functions.

2. Algorithms for calculating IQE according to the modified Katz formula and Ostrom's method for arbitrary order control systems are developed, including vector representation of the objective function of the protection system.

3. The vector penalty function is proposed and the algorithm of its calculation is developed for display of degree of infringement of conditions of stability of parameters of system of protection by Rauss-Hurwitz criterion.

4. Studies have shown the high efficiency of the proposed algorithms. And on the night of February 23-24, 2022, they effectively worked against DDoS attacks carried out by Russian hackers on Ukrainian information systems.

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