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BUILDING A LOGICAL NETWORK FOR SOLVING THE PROBLEM OF CAR RENTAL BY MEANS ALGEBRA OF FINITE PREDICATES

The article is devoted to the research of the tools of algebra of finite predicates for the system analysis and formalization of the task of automating car rental according to the selected parameters. In particular, the process of optimal car selection depends on the following parameters: car class, brand, availability of driver, type of trip and its duration, fuel type, tariff type, as well as season and weather conditions. Each of these criteria has its own area of definition, where you need to take into account all the relationships and influences between the values of the entered variables (criteria). The aim of the work is to increase the speed of data processing in the problem of car rental by dividing the input multi-place ratio into a binary composition. The technique is based on the means and methods of algebra of finite predicates. Introduction of the predicate of object recognition in the specified subject area allowed to formally describe data of any type, and the applied method of construction of logical networks provides increase in speed of information processing due to parallelization of processing processes. Thus, a complex multi-place relation was divided into a composition of binary relations described in the language of predicate algebra, taking into account the detailed system analysis of the subject area. A scientific novelty is the constructed mathematical model of the car rental problem, which is represented by a predicate that depends on thirteen variables. This predicate is characterized by a system of twelve binary relations, which are represented in the article by dual graphs and formulas of the corresponding predicates. The model predicate is a composition of all constructed binary predicates. The practical significance is due to the logical network built on the basis of a mathematical model, which allows from the relationship "many to many" to move to the relationship "to each other" and parallelize the process of information processing. The result is a logical network of car rental problems, which works iteratively until it receives stable results in two consecutive steps and allows you to solve problems of analysis and synthesis for car rental according to selected parameters.

Keywords: algebra of finite predicates, predicate of object recognition, logical network, mathematical model, relation, car rental, subject area, criterion

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ПОБУДОВА ЛОГІЧНОЇ МЕРЕЖІ ДЛЯ РОЗВ'ЯЗАННЯ ЗАДАЧІ ОРЕНДИ АВТОМОБІЛІВ ЗАСОБАМИ АЛГЕБРИ СКІНЧЕННИХ ПРЕДИКАТІВ

Статтю присвячено дослідженню інструментарія алгебри скінченних предикатів для проведення системного аналізу та формалізації задачі автоматизації оренди автомобілів за обраними параметрами. Зокрема, процес оптимального вибору автомобіля залежить від наступних параметрів: класу автомобіля, його марки, наявності водія, типу подорожі та її тривалості, типу палива, типу тарифу, а також сезону та погодних умов. Кожен із зазначених критеріїв має свою область визначення, де потрібно враховувати усі взаємозв'язки та впливи саме між значеннями введених змінних (критеріїв). Метою роботи є підвищення швидкодії обробки данних в задачі оренди автомобілів за рахунок розбиття вхідного багатомісцевого відношення на композицію бінарних. Методика грунтується на засобах та методах алгебри скінченних предикатів. Введення предикату впізнавання предметів на вказаній предметній області дозволило формально описати дані будь-якого типу, а застосований метод побудови логічних мереж забезпечує підвищення швидкості обробки інформації за рахунок розпаралелювання процесів обробки. Таким чином, складне багатомісцеве відношення було розбито на композицію бінарних відношень, що описуються мовою алгебри предикатів з урахуванням деталізованого системного аналізу предметної області. Науковою новизною являється побудована математична модель задачі оренди автомобілів, яку подано предикатом, що залежить від тринадцяти змінних. Цей предикат характеризується системою дванадцяти бінарних відношень, які в статті представлено дводольними графами та формулами відповідних предикатів. Предикат моделі є композицією усіх побудованих бінарних предикатів. Практична значимість обумовлюється побудованою на основі математичної моделі логічною мережею, що дозволяє від відношення «багато до багатьох» перейти до відношень «один до одного» та розпаралелити процес обробки інформації. Результатом роботи є побудована логічна мережа задачі оренди автомобілів, що працює ітераційно до тих пір, поки не отримає сталі результати на двох кроках підряд та дозволяє розв'язувати задачі аналізу та синтезу для оренди автомобілів за вибраними параметрами.

Ключові слова: алгебра скінченних предикатів, предикат упізнавання предметів, логічна мережа, математична модель, відношення, оренда автомобілів, предметна область, критерій.

Introduction

The information processing and transmission, especially in large volumes, accumulates and intersects, which in the wrong organization leads to the loss of some data or to finding the wrong result, the system can also simply be overloaded. Relationships within the system are quite complex [1]. If we consider all the elements of the system to be equivalent and consider their interactions, the structure of the object under study will be too complex for automation and further improvement, as well as for the end user. In addition, the information in the system is often presented in different forms. Therefore, it is important to have the tools to formally describe such data, to describe complex relationships and to build an appropriate mathematical model.

Related works

Nowadays, there are many programs for rent and lease accounting [2-4], designed to automate paperwork, staff work, direct accounting and optimization of various equipment, vehicles, real estate and more. As a rule, they allow you to set the price, fill in the nomenclature, attach a photo, barcode, specify the duration of the lease, even different markups for certain categories of customers. However, little attention is paid (or not taken into account at all) to the fact that the data themselves can be complex, and no direct interdependence between different data is studied, which leads to duplication of information and reduce the speed of its processing.

It is very important to analyze the subject area in detail, and only then build a mathematical model. And to analyze the system, it is advisable to formalize it, identify variables, values of variables and identify the essential relationships.

The language of algebra of finite predicates allows not only to formally describe the process of data processing taking into account the detailed system analysis of the subject area, but also allows to build an economical model in the form of a logical network by decomposing the input many-placed relationship [5,6]. Relationships are described by predicates of algebra of finite predicates, and the introduced predicate of object recognition allows to describe data of any nature [7-9].

Purpose

The problem of automating car rental is solved by building an appropriate logical network by describing the subject area by means of algebra of finite predicates, decomposition of the original multi-place predicate into a binary composition and introduction of additional internal variables characterizing the subject area. The aim of the work is to increase the speed of data processing in the problem of car rental in the form of a logical network by dividing the input multi-place relation into a composition of binary means of algebra of finite predicates.

Proposed technique and Results

The subject area "Car Rental Company" was taken to perform this task. The task consisted of building a logical network using predicate algebra and building a mathematical model.

Regarding the formal description and selection of criteria, three criteria were selected, namely: x_1– car class, x_2 – car name, x_3 – presence of a driver. It should be noted that the attribute of the presence of the driver actually determines the type of service provided (taxi or car sharing).

Regarding the allowable values of the first attribute, it was decided to use six main classes of cars. It should be noted that this classification is generally accepted, but may differ due to certain conditions, for example, if all cars of a certain class are not in sufficient demand. This phenomenon may be caused by insufficient earnings of citizens of the settlement.

x_1^1 – Economy, x_1^2 – Comfort, x_1^3 – Comfort+, x_1^4 – Business, x_1^5 – Premium, x_1^6 – Elite

The company's fleet consists of the following cars:

x_2^1 – KIA Rio, x_2^2 – Chevrolet Lacetti, x_2^3 – Volkswagen Polo, x_2^4 – Hyundai Solaris, x_2^5 – Renault Logan, x_2^6 – Skoda Octavia, x_2^7 – Hyundai Elantra, x_2^8 – Toyota Camri, x_2^9 – Kia Optima, x_2^{10} – Hyundai Sonata, x_2^{11} – Mercedes-Benz E-class, x_2^{12} – BMW 5, x_2^{13} – Audi A6, x_2^{14} – Mercedes-Benz S-class, x_2^{15} – BMW 7, x_2^{16} – Audi A8, x_2^{17} – Mercedes-Maybach S-class

The last attribute can take only two values, and it was decided to order the order of any car listed in the list of available, both with the driver and without him.

The predicate x_3^1 means a presence of a driver, the predicate x_3^2 means a driver is not present

The general view of the car rental company will be as follows (table 1), where the intersection of all attributes instead of a certain value will contain an expression obtained from the algebra of predicates and a pre-specified list of attributes and their valid values.

Table 1

Relationship between class, car name and driver availability

Class	Car Name	With driver	No driver
1	2	3	4
Economy	KIA Rio	$x_1^1 x_2^1 x_3^1 = q_1$	$x_1^1 x_2^1 x_3^2 = q_{18}$
	Chevrolet Lacetti	$x_1^1 x_2^2 x_3^1 = q_2$	$x_1^1 x_2^2 x_3^2 = q_{19}$
	Volkswagen Polo	$x_1^1 x_2^3 x_3^1 = q_3$	$x_1^1 x_2^3 x_3^2 = q_{20}$
	Hyundai Solaris	$x_1^1 x_2^4 x_3^1 = q_4$	$x_1^1 x_2^4 x_3^2 = q_{21}$
	Renault Logan	$x_1^1 x_2^5 x_3^1 = q_5$	$x_1^1 x_2^5 x_3^2 = q_{22}$
Comfort	Skoda Octavia	$x_1^2 x_2^6 x_3^1 = q_6$	$x_1^2 x_2^6 x_3^2 = q_{23}$
	Hyundai Elantra	$x_1^2 x_2^7 x_3^1 = q_7$	$x_1^2 x_2^7 x_3^2 = q_{24}$
Comfort+	Toyota Camri	$x_1^3 x_2^8 x_3^1 = q_8$	$x_1^3 x_2^8 x_3^2 = q_{25}$
	Kia Optima	$x_1^3 x_2^9 x_3^1 = q_9$	$x_1^3 x_2^9 x_3^2 = q_{26}$
	Hyundai Sonata	$x_1^3 x_2^{10} x_3^1 = q_{10}$	$x_1^3 x_2^{10} x_3^2 = q_{27}$

1	2	3	4
Business	Mercedes-Benz E-class	$x_1^4x_2^{11}x_3^1=q_{11}$	$x_1^4x_2^{11}x_3^2=q_{28}$
	BMW 5	$x_1^4x_2^{12}x_3^1=q_{12}$	$x_1^4x_2^{12}x_3^2=q_{29}$
	Audi A6	$x_1^4x_2^{13}x_3^1=q_{13}$	$x_1^4x_2^{13}x_3^2=q_{30}$
Premium	Mercedes-Benz S-class	$x_1^5x_2^{14}x_3^1=q_{14}$	$x_1^5x_2^{14}x_3^2=q_{31}$
	BMW 7	$x_1^5x_2^{15}x_3^1=q_{15}$	$x_1^5x_2^{15}x_3^2=q_{32}$
	Audi A8	$x_1^5x_2^{16}x_3^1=q_{16}$	$x_1^5x_2^{16}x_3^2=q_{33}$
Elite	Mercedes-Maybach S-class	$x_1^6x_2^{17}x_3^1=q_{17}$	$x_1^6x_2^{17}x_3^2=q_{34}$

Next you need to use the variable m – determination of the name of the car according to its class and the presence of the driver. To define m you must consider the values of all possible combinations of attributes x_1, x_2 and x_3. In this case the predicate of the car name will look like this:

$$m(x_1,x_2,x_3)= x_1^1x_2^1x_3^1 \vee x_1^1x_2^1x_3^2 \vee x_1^1x_2^2x_3^1 \vee x_1^1x_2^2x_3^2 \vee x_1^1x_2^3x_3^1 \vee x_1^1x_2^3x_3^2 \vee x_1^1x_2^4x_3^1 \vee x_1^1x_2^4x_3^2 \vee x_1^1x_2^5x_3^1 \vee x_1^1x_2^5x_3^2 \vee x_1^2x_2^6x_3^1 \vee x_1^2x_2^6x_3^2 \vee x_1^2x_2^7x_3^1 \vee x_1^2x_2^7x_3^2 \vee x_1^3x_2^8x_3^1 \vee x_1^3x_2^8x_3^2 \vee x_1^3x_2^9x_3^1 \vee x_1^3x_2^9x_3^2 \vee x_1^3x_2^{10}x_3^1 \vee x_1^3x_2^{10}x_3^2 \vee x_1^4x_2^{11}x_3^1 \vee x_1^4x_2^{11}x_3^2 \vee x_1^4x_2^{12}x_3^1 \vee x_1^4x_2^{12}x_3^2 \vee x_1^4x_2^{13}x_3^1 \vee x_1^4x_2^{13}x_3^2 \vee x_1^5x_2^{14}x_3^1 \vee x_1^5x_2^{14}x_3^2 \vee x_1^5x_2^{15}x_3^1 \vee x_1^5x_2^{15}x_3^2 \vee x_1^5x_2^{16}x_3^1 \vee x_1^5x_2^{16}x_3^2 \vee x_1^6x_2^{17}x_3^1 \vee x_1^6x_2^{17}x_3^2.$$

Performing a disjunction operation on a variable:

$$x_1^1x_2^1x_3^1 \vee x_1^1x_2^1x_3^2 = q_1 \vee q_{18} = m_1,$$
$$x_1^1x_2^2x_3^1 \vee x_1^1x_2^2x_3^2 = q_2 \vee q_{19} = m_2,$$
$$x_1^1x_2^3x_3^1 \vee x_1^1x_2^3x_3^2 = q_3 \vee q_{20} = m_3,$$
$$x_1^1x_2^4x_3^1 \vee x_1^1x_2^4x_3^2 = q_4 \vee q_{21} = m_4,$$
$$x_1^1x_2^5x_3^1 \vee x_1^1x_2^5x_3^2 = q_5 \vee q_{22} = m_5,$$
$$x_1^2x_2^6x_3^1 \vee x_1^2x_2^6x_3^2 = q_6 \vee q_{23} = m_6,$$
$$x_1^2x_2^7x_3^1 \vee x_1^2x_2^7x_3^2 = q_7 \vee q_{24} = m_7,$$
$$x_1^3x_2^8x_3^1 \vee x_1^3x_2^8x_3^2 = q_8 \vee q_{25} = m_8,$$
$$x_1^3x_2^9x_3^1 \vee x_1^3x_2^9x_3^2 = q_9 \vee q_{26} = m_9,$$
$$x_1^3x_2^{10}x_3^1 \vee x_1^3x_2^{10}x_3^2 = q_{10} \vee q_{27} = m_{10},$$
$$x_1^4x_2^{11}x_3^1 \vee x_1^4x_2^{11}x_3^2 = q_{11} \vee q_{28} = m_{11},$$
$$x_1^4x_2^{12}x_3^1 \vee x_1^4x_2^{12}x_3^2 = q_{12} \vee q_{29} = m_{12},$$
$$x_1^4x_2^{13}x_3^1 \vee x_1^4x_2^{13}x_3^2 = q_{13} \vee q_{30} = m_{13},$$
$$x_1^5x_2^{14}x_3^1 \vee x_1^5x_2^{14}x_3^2 = q_{14} \vee q_{31} = m_{14},$$
$$x_1^5x_2^{15}x_3^1 \vee x_1^5x_2^{15}x_3^2 = q_{15} \vee q_{32} = m_{15},$$
$$x_1^5x_2^{16}x_3^1 \vee x_1^5x_2^{16}x_3^2 = q_{16} \vee q_{33} = m_{16},$$
$$x_1^6x_2^{17}x_3^1 \vee x_1^6x_2^{17}x_3^2 = q_{17} \vee q_{34} = m_{17}.$$

The motive that prompted the operation of a disjunction operation on a variable is the desire to obtain an economical system of influences of definitions, in which each influence of the definition of the name of the car would correspond to one and only one name.

Binarization of a predicate that combines m with variables x_1, x_2 and x_3

$$P_1(x_1, m) = x_1^1(m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5) \vee x_1^2(m_6 \vee m_7) \vee x_1^3(m_8 \vee m_9 \vee m_{10}) \vee x_1^4(m_{11} \vee m_{12} \vee m_{13}) \vee x_1^5(m_{14} \vee m_{15} \vee m_{16}) \vee x_1^6 m_{17},$$
$$P_2(x_2, m) = x_2^1m_1 \vee x_2^2m_2 \vee x_2^3m_3 \vee x_2^4m_4 \vee x_2^5m_5 \vee x_2^6m_6 \vee x_2^7m_7 \vee x_2^8m_8 \vee x_2^9m_9 \vee x_2^{10}m_{10} \vee x_2^{11}m_{11} \vee x_2^{12}m_{12} \vee x_2^{13}m_{13} \vee x_2^{14}m_{14} \vee x_2^{15}m_{15} \vee x_2^{16}m_{16} \vee x_2^{17}m_{17},$$
$$P_3(x_3, m) = (x_3^1 \vee x_3^2)(m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6 \vee m_7 \vee m_8 \vee m_9 \vee m_{10} \vee m_{11} \vee m_{12} \vee m_{13} \vee m_{14} \vee m_{15} \vee m_{16} \vee m_{17}).$$

Let's represent the relations obtained by binarizing the predicate using graphs.

Graph of the relationship between m and x_1 variables shown on fig.1.

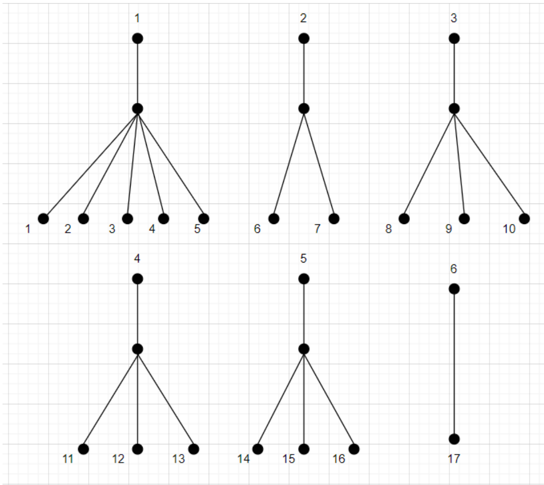

Fig.1. Graph of the relationship between m and x_1 variables

Graph of the relationship between m and x_2 variables shown on fig.2, where N belongs to the range from 1 to 17 inclusive.

Fig.2. Graph of the relationship between m and x_2 variables

Graph of the relationship between m and x_3 variables shown on fig.3.

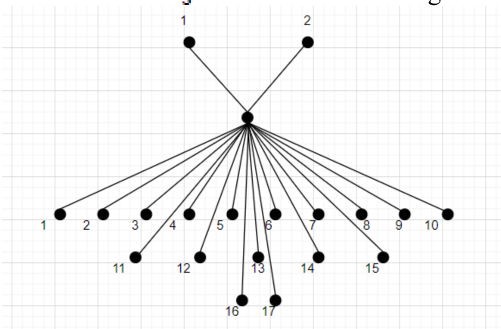

Fig.3. Graph of the relationship between m and x_3 variables

Let's classify further the cars available for rent according to the following criteria: y_1– rate type, y_2 – trip type.

According to the first attribute, rates are divided into: y_1^1 – morning, y_1^2 – day, y_1^3 – evening, y_1^4 – night, y_1^5 – mixed, for the second attribute, it can take the following values: y_2^1 – trip through the city, y_2^2 – trip between towns.

Table 2

Relationship between rate type and trip type

Rate type	Trip through the city	Trip between towns
morning	$y_1^1 y_2^1 = k_1$	$y_1^1 y_2^2 = k_6$
day	$y_1^2 y_2^1 = k_2$	$y_1^2 y_2^2 = k_7$
Evening	$y_1^3 y_2^1 = k_3$	$y_1^3 y_2^2 = k_8$
Night	$y_1^4 y_2^1 = k_4$	$y_1^4 y_2^2 = k_9$
Mixed	$y_1^5 y_2^1 = k_5$	$y_1^5 y_2^2 = k_{10}$

Next you need to use the variable n – determination the name of the car according to the type of available rate and trip. To determine n you must consider the values of all possible combinations of attributes $y_1$and y_2. In this case the predicate of the car name will look like this:

$$n\,(y_1, y_2) = y_1^1 y_2^1 \vee y_1^1 y_2^2 \vee y_1^2 y_2^1 \vee y_1^2 y_2^2 \vee y_1^3 y_2^1 \vee y_1^3 y_2^2 \vee y_1^4 y_2^1 \vee y_1^4 y_2^2 \vee y_1^5 y_2^1 \vee y_1^5 y_2^2.$$

Performing a disjunction operation on a variable:

$$y_1^1y_2^1 \vee y_1^1y_2^2 = k_1 \vee k_6 = n_1,$$
$$y_1^2y_2^1 \vee y_1^2y_2^2 = k_2 \vee k_7 = n_2,$$
$$y_1^3y_2^1 \vee y_1^3y_2^2 = k_3 \vee k_8 = n_3,$$
$$y_1^4y_2^1 \vee y_1^4y_2^2 = k_4 \vee k_9 = n_4,$$
$$y_1^5y_2^1 \vee y_1^5y_2^2 = k_5 \vee k_{10} = n_5.$$

Let's binarize the predicate that combines n with variables $y_1$and y_2:

$$P_4(y_1, n) = y_1^1n_1 \vee y_1^2n_2 \vee y_1^3n_3 \vee y_1^4n_4 \vee y_1^5n_5,$$
$$P_5(y_2, n) = (y_2^1 \vee y_2^2)(n_1 \vee n_2 \vee n_3 \vee n_4 \vee n_5).$$

Let's represent the relations received at binarization of a predicate using graphs.

Graph of the relationship between n and y_1 variables shown on fig.4, where N belongs to the range from 1 to 5 inclusive.

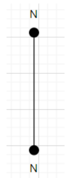

Fig.4. Graph of the relationship between n and y_1 variables

Graph of the relationship between n and y_2 variables shown on fig. 5.

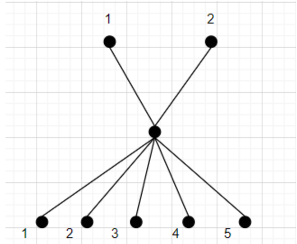

Fig.5. Graph of the relationship between n and $y_2$variables

In the case of long trips or long-term car rental, cars should be classified according to the following criteria: z_1– a fuel type that is accepted by car, z_2 – a duration of trip with a full tank (table 3).

According to the first attribute, its allowable values are the three most popular fuels, namely: z_1^1 – petrol, z_1^2 – electricity, z_1^3 – diesel. As for the second attribute, it was decided to divide it into the following values: z_2^1 – up to 2 hours, z_2^2 –up to 6 hours, z_2^3 – until a day, z_2^4 – more than a day.

Table 3

Relationship between class, car name and driver availability

Trip duration	Fuel type		
	Petrol	Electricity	Diesel
Up to 2 hours	$z_1^1z_2^1 = l_1$	$z_1^2z_2^1 = l_5$	–
Up to 6 hours	$z_1^1z_2^2 = l_2$	$z_1^2z_2^2 = l_6$	$z_1^3z_2^2 = l_9$
Until a Day	$z_1^1z_2^3 = l_3$	$z_1^2z_2^3 = l_7$	$z_1^3z_2^3 = l_{10}$
More than a day	$z_1^1z_2^4 = l_4$	$z_1^2z_2^4 = l_8$	–

Next, you need to use the variable b – determining the name of the car according to the type of fuel and duration of the trip. To determine b you must consider the values of all possible combinations of attributes z_1 and z_2. In this case the predicate of the car name will look like this:

$$b\,(z_1, z_2) = z_1^1z_2^1 \vee z_1^1z_2^2 \vee z_1^1z_2^3 \vee z_1^1z_2^4 \vee z_1^2z_2^1 \vee z_1^2z_2^2 \vee z_1^2z_2^3 \vee z_1^2z_2^4 \vee z_1^3z_2^2 \vee z_1^3z_2^3.$$

Performing a disjunction operation on a variable:

$$z_1^1z_2^1 \vee z_1^2z_2^1 = l_1 \vee l_5 = b_1,$$
$$z_1^1z_2^2 \vee z_1^2z_2^2 \vee z_1^3z_2^2 = l_2 \vee l_6 \vee l_9 = b_2,$$
$$z_1^1z_2^3 \vee z_1^2z_2^3 \vee z_1^3z_2^3 = l_3 \vee l_7 \vee l_{10} = b_3,$$

$$z_1^1 z_2^4 \vee z_1^2 z_2^4 = l_4 \vee l_8 = b_4.$$

Let's binarize the predicate that combines b with z_1та z_2 variables:

$$P_6(z_1, b) = (z_1^1 \vee z_1^2)(b_1 \vee b_2 \vee b_3 \vee b_4) \vee z_1^3(b_2 \vee b_3),$$
$$P_7(z_2, b) = z_2^1 b_1 \vee z_2^2 b_2 \vee z_2^3 b_3 \vee z_2^4 b_4 .$$

Let's represent the relations received at binarization of a predicate using graphs.
Graph of the relationship between b and z_1 variables shown on fig. 6.

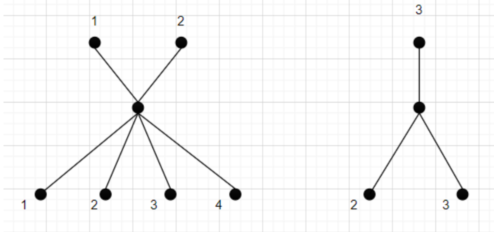

Fig.6. Graph of the relationship between n and y_2 variables

Graph of the relationship between b and z_2 variables shown on fig. 7, where N belongs to the range from 1 to 4 inclusive.

Fig.7. Graph of the relationship between b and z_2 variables

In order to increase the safety of our customers and eliminate unwanted traffic accidents while driving, it was necessary to classify all cars according to the following criteria: r_1– season, r_2 – weather conditions (table 4).

According to the first attribute, its allowable values are the three most popular fuels, namely: r_1^1 – summer, r_1^2 – autumn, r_1^3 – winter, r_1^4 – spring. Regarding the second attribute, it was decided to take into account the following weather conditions: r_2^1 – fog, r_2^2 – rain, r_2^3 – wind, r_2^4 – snow, r_2^5 – sleet.

Table 4

Relationship between season and weather conditions

Weather conditions	Season			
	Summer	Autumn	Winter	Spring
Fog	$r_1^1 r_2^1 = p_1$	$r_1^2 r_2^1 = p_4$	$r_1^3 r_2^1 = p_7$	$r_1^4 r_2^1 = p_{11}$
Rain	$r_1^1 r_2^2 = p_2$	$r_1^2 r_2^2 = p_5$	–	$r_1^4 r_2^2 = p_{12}$
Wind	$r_1^1 r_2^3 = p_3$	$r_1^2 r_2^3 = p_6$	$r_1^3 r_2^3 = p_8$	$r_1^4 r_2^3 = p_{13}$
Snow	–	–	$r_1^3 r_2^4 = p_9$	–
Sleet	–	–	$r_1^3 r_2^5 = p_{10}$	–

Next you need to use the variable c – determination of car name according to season and weather conditions. To determine c you must consider the values of all possible attribute combinations r_1 and r_2. In this case the predicate of car name will look like this:

$$c(r_1, r_2) = r_1^1 r_2^1 \vee r_1^1 r_2^2 \vee r_1^1 r_2^3 \vee r_1^2 r_2^1 \vee r_1^2 r_2^2 \vee r_1^2 r_2^3 \vee r_1^3 r_2^1 \vee r_1^3 r_2^3 \vee r_1^3 r_2^4 \vee r_1^3 r_2^5 \vee$$
$$r_1^4 r_2^1 \vee r_1^4 r_2^2 \vee r_1^4 r_2^3.$$

Performing a disjunction operation on a variable:

$$r_1^1 r_2^1 \vee r_1^2 r_2^1 \vee r_1^3 r_2^1 \vee r_1^4 r_2^1 = p_1 \vee p_4 \vee p_7 \vee p_{11} = c_1,$$
$$r_1^1 r_2^2 \vee r_1^2 r_2^2 \vee r_1^4 r_2^2 = p_2 \vee p_5 \vee p_{12} = c_2,$$
$$r_1^1 r_2^3 \vee r_1^2 r_2^3 \vee r_1^3 r_2^3 \vee r_1^4 r_2^3 = p_3 \vee p_6 \vee p_8 \vee p_{13} = c_3,$$
$$r_1^3 r_2^4 = p_9 = c_4,$$
$$r_1^3 r_2^5 = p_{10} = c_5.$$

Let's binarize the connecting predicate c with $r_1$and r_2 variables:

$$P_8(r_1, c) = (r_1^1 \vee r_1^2 \vee r_1^4)(c_1 \vee c_2 \vee c_3) \vee r_1^3(c_1 \vee c_3 \vee c_4 \vee c_5),$$
$$P_9(r_2, c) = r_2^1 c_1 \vee r_2^2 c_2 \vee r_2^3 c_3 \vee r_2^4 c_4 \vee r_2^5 c_5.$$

Let's represent the relations received at binarization of a predicate using graphs.

Graph of the relationship between c and r_1 variables shown on fig. 8.

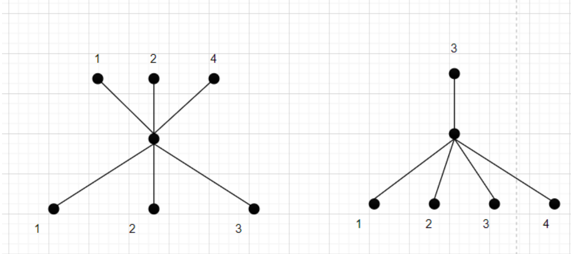

Fig.8. Graph of the relationship between c and r_1 variables

Graph of the relationship between c and r_2 variables shown on fig. 9, where N belongs a range from 1 to 5 exclusive.

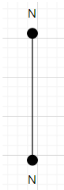

Fig.9. Graph of the relationship between c and r_2 variables

Let's build a mathematical model for determining the car in relation to its characteristics by class and car name, as well as rate.

A paradigmatic relationship table between m and n displayed in Table 5.

Table 5

Relationship between car class, name and rate

Class	Name	Rate				
		Morning	Day	Evening	Night	Mixed
Economy	KIA Rio	$x_1^1x_2^1y_1^1$	$x_1^1x_2^1y_1^2$	$x_1^1x_2^1y_1^3$	$x_1^1x_2^1y_1^4$	$x_1^1x_2^1y_1^5$
	Chevrolet Lacetti	$x_1^1x_2^2y_1^1$	$x_1^1x_2^2y_1^2$	$x_1^1x_2^2y_1^3$	$x_1^1x_2^2y_1^4$	$x_1^1x_2^2y_1^5$
	Volkswagen Polo	$x_1^1x_2^3y_1^1$	$x_1^1x_2^3y_1^2$	$x_1^1x_2^3y_1^3$	$x_1^1x_2^3y_1^4$	$x_1^1x_2^3y_1^5$
	Hyundai Solaris	$x_1^1x_2^4y_1^1$	$x_1^1x_2^4y_1^2$	$x_1^1x_2^4y_1^3$	$x_1^1x_2^4y_1^4$	$x_1^1x_2^4y_1^5$
	Renault Logan	$x_1^1x_2^5y_1^1$	$x_1^1x_2^5y_1^2$	$x_1^1x_2^5y_1^3$	$x_1^1x_2^5y_1^4$	$x_1^1x_2^5y_1^5$
Comfort	Skoda Octavia	$x_1^2x_2^6y_1^1$	$x_1^2x_2^6y_1^2$	$x_1^2x_2^6y_1^3$	$x_1^2x_2^6y_1^4$	$x_1^2x_2^6y_1^5$
	Hyundai Elantra	$x_1^2x_2^7y_1^1$	$x_1^2x_2^7y_1^2$	$x_1^2x_2^7y_1^3$	$x_1^2x_2^7y_1^4$	$x_1^2x_2^7y_1^5$
Comfort+	Toyota Camri	$x_1^3x_2^8y_1^1$	$x_1^3x_2^8y_1^2$	$x_1^3x_2^8y_1^3$	$x_1^3x_2^8y_1^4$	-
	Kia Optima	$x_1^3x_2^9y_1^1$	$x_1^3x_2^9y_1^2$	$x_1^3x_2^9y_1^3$	-	$x_1^3x_2^9y_1^5$
	Hyundai Sonata	-	$x_1^3x_2^{10}y_1^2$	$x_1^3x_2^{10}y_1^3$	$x_1^3x_2^{10}y_1^4$	$x_1^3x_2^{10}y_1^5$
Business	Mercedes-Benz E-class	$x_1^4x_2^{11}y_1^1$	$x_1^4x_2^{11}y_1^2$	$x_1^4x_2^{11}y_1^3$	-	-
	BMW 5	$x_1^4x_2^{12}y_1^1$	$x_1^4x_2^{12}y_1^2$	$x_1^4x_2^{12}y_1^3$	$x_1^4x_2^{12}y_1^4$	$x_1^4x_2^{12}y_1^5$
	Audi A6	$x_1^4x_2^{13}y_1^1$	$x_1^4x_2^{13}y_1^2$	$x_1^4x_2^{13}y$	-	-
Premium	Mercedes-Benz S-class	-	$x_1^5x_2^{14}y_1^2$	$x_1^5x_2^{14}y_1^2$	-	$x_1^5x_2^{14}y_1^2$
	BMW 7	$x_1^5x_2^{15}y_1^1$	$x_1^5x_2^{15}y_1^2$	$x_1^5x_2^{15}y_1^3$	-	-
	Audi A8	$x_1^5x_2^{16}y_1^1$	$x_1^5x_2^{16}y_1^2$	$x_1^5x_2^{16}y_1^3$	$x_1^5x_2^{16}y_1^4$	-
Elite	Mercedes-Maybach S-class	-	$x_1^6x_2^{17}y_1^2$	$x_1^6x_2^{17}y_1^3$	-	$x_1^6x_2^{17}y_1^4$

We binarize:

$$P_{10}(x_1, x_2, y_1) = y_1^1(x_1^1x_2^1 \vee x_1^1x_2^2 \vee x_1^1x_2^3 \vee x_1^1x_2^4 \vee x_1^1x_2^5 \vee x_1^2x_2^6 \vee x_1^2x_2^7 \vee x_1^3x_2^8 \vee x_1^3x_2^9 \vee x_1^4x_2^{11} \vee x_1^4x_2^{12} \vee x_1^4x_2^{13} \vee x_1^5x_2^{15} \vee x_1^5x_2^{16}) \vee y_1^2(x_1^1x_2^1 \vee x_1^1x_2^2 \vee x_1^1x_2^3 \vee x_1^1x_2^4 \vee x_1^1x_2^5 \vee x_1^2x_2^6 \vee x_1^2x_2^7 \vee x_1^3x_2^8 \vee x_1^3x_2^9 \vee x_1^3x_2^{10} \vee x_1^4x_2^{11} \vee x_1^4x_2^{12} \vee x_1^4x_2^{13} \vee x_1^5x_2^{14} \vee x_1^5x_2^{15} \vee x_1^5x_2^{16} \vee x_1^6x_2^{17}) \vee y_1^3(x_1^1x_2^1 \vee x_1^1x_2^2 \vee x_1^1x_2^3 \vee x_1^1x_2^4 \vee x_1^1x_2^5 \vee x_1^2x_2^6 \vee x_1^2x_2^7 \vee x_1^3x_2^8 \vee x_1^3x_2^9 \vee x_1^3x_2^{10} \vee x_1^4x_2^{11} \vee x_1^4x_2^{12} \vee x_1^4x_2^{13} \vee x_1^5x_2^{14} \vee x_1^5x_2^{15} \vee x_1^5x_2^{16} \vee x_1^6x_2^{17}) \vee y_1^4(x_1^1x_2^1 \vee x_1^1x_2^2 \vee x_1^1x_2^3 \vee x_1^1x_2^4 \vee x_1^1x_2^5 \vee x_1^2x_2^6 \vee x_1^2x_2^7 \vee x_1^3x_2^8 \vee x_1^3x_2^{10} \vee x_1^4x_2^{12} \vee x_1^5x_2^{16}) \vee y_1^5(x_1^1x_2^1 \vee x_1^1x_2^2 \vee x_1^1x_2^3 \vee x_1^1x_2^4 \vee x_1^1x_2^5 \vee x_1^2x_2^6 \vee x_1^2x_2^7 \vee x_1^3x_2^9 \vee x_1^3x_2^{10} \vee x_1^4x_2^{12} \vee x_1^5x_2^{14} \vee x_1^6x_2^{17}).$$

Let's build a mathematical model for determining the car in relation to its characteristics by trip type and car name, as well as fuel type.

Paradigmatic table of relationships between entered intermediate m and b variables shown Table 6

Table 6

Relationship between trip type, car name and fuel type

Name	With driver			Without driver		
	Petrol	Electricity	Diesel	Petrol	Electricity	Diesel
KIA Rio	$x_2^1 x_3^1 z_1^1$	–	–	$x_2^1 x_3^2 z_1^1$	–	–
Chevrolet Lacetti	$x_2^2 x_3^1 z_1^1$	–	–	$x_2^2 x_3^2 z_1^1$	–	–
Volkswagen Polo	–	–	$x_2^3 x_3^1 z_1^3$	–	–	$x_2^3 x_3^2 z_1^3$
Hyundai Solaris	–	–	$x_2^4 x_3^1 z_1^3$	–	–	$x_2^4 x_3^2 z_1^3$
Renault Logan	$x_2^5 x_3^1 z_1^1$	–	–	$x_2^5 x_3^2 z_1^1$	–	–
Skoda Octavia	$x_2^6 x_3^1 z_1^1$	–	–	$x_2^6 x_3^2 z_1^1$	–	–
Hyundai Elantra	–	$x_2^7 x_3^1 z_1^2$	–	–	$x_2^7 x_3^2 z_1^2$	–
Toyota Camri	–	–	$x_2^8 x_3^1 z_1^3$	–	–	$x_2^8 x_3^2 z_1^3$
Kia Optima	$x_2^9 x_3^1 z_1^1$	–	–	$x_2^9 x_3^2 z_1^1$	–	–
Hyundai Sonata	–	–	$x_2^{10} x_3^1 z_1^3$	–	–	$x_2^{10} x_3^2 z_1^3$
Mercedes-Benz E-class	$x_2^{11} x_3^1 z_1^1$	–	–	$x_2^{11} x_3^2 z_1^1$	–	–
BMW 5	$x_2^{12} x_3^1 z_1^1$	–	–	$x_2^{12} x_3^2 z_1^1$	–	–
Audi A6	–	$x_2^{13} x_3^1 z_1^2$	–	–	$x_2^{13} x_3^2 z_1^2$	–
Mercedes-Benz S-class	–	$x_2^{14} x_3^1 z_1^2$	–	–	$x_2^{14} x_3^2 z_1^2$	–
BMW 7	–	–	$x_2^{15} x_3^1 z_1^3$	–	–	$x_2^{15} x_3^2 z_1^3$
Audi A8	–	$x_2^{16} x_3^1 z_1^2$	–	–	$x_2^{16} x_3^2 z_1^2$	–
Mercedes-Maybach S-class	$x_2^{17} x_3^1 z_1^1$	–	–	$x_2^{17} x_3^2 z_1^1$	–	–

We binarize:

$$P_{11}(x_2, x_3, z_1) =$$

$$z_1^1 (x_2^1 x_3^1 \vee x_2^2 x_3^1 \vee x_2^5 x_3^1 \vee x_2^6 x_3^1 \vee x_2^9 x_3^1 \vee x_2^{11} x_3^1 \vee x_2^{12} x_3^1 \vee x_2^{17} x_3^1 \vee x_2^1 x_3^2 \vee x_2^2 x_3^2 \vee x_2^5 x_3^2 \vee x_2^6 x_3^2 \vee x_2^9 x_3^2 \vee x_2^{11} x_3^2 \vee x_2^{12} x_3^2 \vee x_2^{17} x_3^2)$$

$$\vee z_1^2 (x_2^7 x_3^1 \vee x_2^{13} x_3^1 \vee x_2^{14} x_3^1 \vee x_2^{16} x_3^1 \vee x_2^7 x_3^2 \vee x_2^{13} x_3^2 \vee x_2^{14} x_3^2 \vee x_2^{16} x_3^2) \vee z_1^3 (x_2^3 x_3^1 \vee x_2^4 x_3^1 \vee x_2^8 x_3^1 \vee x_2^{10} x_3^1 \vee x_2^{15} x_3^1 \vee x_2^3 x_3^2 \vee x_2^4 x_3^2 \vee x_2^8 x_3^2 \vee x_2^{10} x_3^2 \vee x_2^{15} x_3^2)$$

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Let's build a mathematical model for determining the car in relation to its characteristics by trip type and rate type, as well as weather conditions.

Paradigmatic table of relationships between entered intermediate n and c variables shown on Table 7.

Table 7

Relationship between travel type, rate type and weather conditions

Rate type	Weather conditions	In town	Between cities
1	2	3	4
Morning	Fog	$y_1^1 y_2^1 r_2^1$	$y_1^1 y_2^2 r_2^1$
	Rain	$y_1^1 y_2^1 r_2^2$	$y_1^1 y_2^2 r_2^2$
	Wind	$y_1^1 y_2^1 r_2^3$	$y_1^1 y_2^2 r_2^3$
	Snow	$y_1^1 y_2^1 r_2^4$	$y_1^1 y_2^2 r_2^4$
	Sleet	$y_1^1 y_2^1 r_2^5$	$y_1^1 y_2^2 r_2^5$
Day	Fog	$y_1^2 y_2^1 r_2^1$	$y_1^2 y_2^2 r_2^1$
	Rain	$y_1^2 y_2^1 r_2^2$	$y_1^2 y_2^2 r_2^2$
	Wind	$y_1^2 y_2^1 r_2^3$	$y_1^2 y_2^2 r_2^3$
	Snow	$y_1^2 y_2^1 r_2^4$	$y_1^2 y_2^2 r_2^4$
	Sleet	$y_1^2 y_2^1 r_2^5$	$y_1^2 y_2^2 r_2^5$
Evening	Fog	$y_1^3 y_2^1 r_2^1$	$y_1^3 y_2^2 r_2^1$
	Rain	$y_1^3 y_2^1 r_2^2$	$y_1^3 y_2^2 r_2^2$
	Wind	$y_1^3 y_2^1 r_2^3$	$y_1^3 y_2^2 r_2^3$
	Snow	$y_1^3 y_2^1 r_2^4$	$y_1^3 y_2^2 r_2^4$
	Sleet	$y_1^3 y_2^1 r_2^5$	$y_1^3 y_2^2 r_2^5$

1	2	3	4
Night	Fog	$y_1^4 y_2^1 r_2^1$	$y_1^4 y_2^2 r_2^1$
	Rain	$y_1^4 y_2^1 r_2^2$	$y_1^4 y_2^2 r_2^2$
	Wind	$y_1^4 y_2^1 r_2^3$	$y_1^4 y_2^2 r_2^3$
	Snow	$y_1^4 y_2^1 r_2^4$	$y_1^4 y_2^2 r_2^4$
	Sleet	$y_1^4 y_2^1 r_2^5$	$y_1^4 y_2^2 r_2^5$
Mixed	Fog	$y_1^5 y_2^1 r_2^1$	$y_1^5 y_2^2 r_2^1$
	Rain	$y_1^5 y_2^1 r_2^2$	$y_1^5 y_2^2 r_2^2$
	Wind	$y_1^5 y_2^1 r_2^3$	$y_1^5 y_2^2 r_2^3$
	Snow	$y_1^5 y_2^1 r_2^4$	$y_1^5 y_2^2 r_2^4$
	Sleet	–	–

We binarize:

$$P_{12}(y_1, y_2, r_2) =$$
$$(r_2^1 \vee r_2^2 \vee r_2^3 \vee r_2^4)(y_1^1 y_2^1 \vee y_1^2 y_2^1 \vee y_1^3 y_2^1 \vee y_1^4 y_2^1 \vee y_1^5 y_2^1 \vee y_1^1 y_2^2 \vee y_1^2 y_2^2 \vee y_1^3 y_2^2 \vee y_1^4 y_2^2 \vee y_1^5 y_2^2)$$
$$\vee r_2^5 (y_1^1 y_2^1 \vee y_1^2 y \vee y_1^3 y_2^1 \vee y_1^4 y_2^1 \vee y_1^1 y_2^2 \vee y_1^2 y_2^2 \vee y_1^3 y_2^2 \vee y_1^4 y_2^2).$$

Conclusions

Thus, a mathematical model was built in the course of the work. This model combines certain characteristics and with the help of this model it is possible to accurately determine the attribute – car name. The multi-place predicate, which reflected all the selection criteria (9 variables) by the user for car rental, is presented in the form of a composition of binary:

$$P(x_1, x_2, x_3, m, y_1, y_2, n_1, r_1, r_2, c, z_1, z_2, b) = P_1(x_1, m) \wedge P_2(x_2, m) \wedge P_3(x_3, m) \wedge P_4(y_1, n) \wedge$$
$$\wedge P_5(y_2, n) \wedge P_6(z_1, b) \wedge P_7(z_2, b) \wedge P_8(r_1, c) \wedge P_9(r_2, c) \wedge P_{10}(m, b) \wedge P_{11}(n, c) \wedge P_{12}(m, n).$$

We see that the constructed mathematical model in the form of a predicate P depends on 13 variables: 9 selected parameters and 4 intermediate variables. It is the introduction of these intermediate variables that made it possible to break the multi-place relation into binary ones. The complexity of the method of building a logical network is precisely the analysis of the system for optimal input of intermediate variables. To solve this problem, it was necessary to systematize all parameters in the form of tables, write the values obtained by the formulas of algebra of finite predicates through predicates of object recognition, then write the predicates corresponding to paradigmatic tables, and conduct, where possible, disjunctive gluing operation, binarization.

The results and data obtained during the work can be taken to the next display of the logical network (fig. 10):

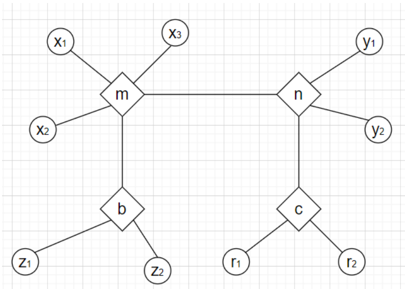

Fig.10. Logical car rental network

It should be noted that in addition to network performance, due to the parallelization of processes, it is important to be able to solve the network not only analysis problems, but also synthesis problems [10-11]. That is, the network can not only issue a car brand (or some other unknown parameter) based on the values entered in the nodes, but, working iteratively, allows you to set the value on all nodes and determine its truth, or, conversely, the known value of one of the parameters, find all relevant possible solutions.

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