MATHEMATICAL MODEL DEPENDENCIES BAUD RATE FROM THE LENGTH OF THE SPECIALIZED DIGITAL NETWORK CANOPEN

At modern enterprises, the main increase in production occurs due to the increase in labor productivity by automating the processes of managing technological processes. Modern automation systems for the control of technological processes are, as a rule, created on the basis of microprocessor elements and specialized digital networks. Many modern process automation systems for controlling technological processes are based on a specialized CANopen digital network. The maximum length of a CANopen network segment can be 5000 m. The data transfer rate decreases as the segment length increases. However, until now there was no mathematical model of the change in data transmission speed depending on the length of the network segment. Therefore, the developers of the network and the organizations that support it experimentally determined the value of data transmission speed on network segments with a length of: 25, 50, 100, 250, 500, 1000, 2500, 5000 m. This fact complicated the process of analyzing computer systems at the stage their design. During the design of computer systems for control and management of industrial facilities that require a guaranteed reaction to random events in a set period of time, developers, as a rule, in cases where the length of the segment is in the intermediate sections between the specified points, perform calculations on the value of the speed for longer segment length than is real. Considering that there may be tens of thousands of nodes on complex objects, the sum results in a significant deviation from real performance indicators in the direction of increasing costs for unnecessary increase in system performance due to the impossibility of using in calculations the real values of the data transmission speed for the real length of the segment between a node that transmits data and that receives data. A mathematical model of the dependence of the data transfer rate on the length of the CANopen network segment has been developed. Such a mathematical model should ensure the determination of the data transfer rate at any point of the segment of the specialized CANopen digital network. This will contribute to the improvement of the quality of design and will allow to create computer control and management systems that will meet the requirements of the consumer without excessive overspending.

Keywords: specialized digital network, specialized computer network, fieldbus, mathematical model, CANopen

Introduction

In modern enterprises, the main increase in production is due to the increase in productivity through the automation of process control processes [1, 2].

Modern process automation systems are usually built on the basis of microprocessor elements and specialized computer networks. Many modern process automation systems are based on the specialized digital network CANopen [2-4]. The maximum length of the network CANopen can be 5,000 m. Data transmission speed decreases with increasing length of the specialized digital network CANopen. However, there is currently no mathematical model for
changing the baud rate depending length of the network. Therefore, network designers and organizations that support it have experimentally set the baud rates for segments with lengths of 25, 50, 100, 250, 500, 1000, 2500, 5000 m [5]. The above fact complicates the process of analyzing computer systems at the design stage. The baud rate is one of the main in this analysis, and for this network it is known only at a few points from 5000 m. Therefore the typical task of the analysis - the performance evaluation of the developed system is much complicated and can not be performed qualitatively. When designing computer systems for the control and management of industrial sites that require a guaranteed response to random events within a fixed period of time, developers typically calculate the velocity values for intermediate segments between specified points as for longer segment length than is real. Considering that there can be tens of thousands of nodes in complex objects, the sum deviates significantly from the real performance indicators in the direction of increasing the costs of unnecessary improvement of system performance due to the inability to use in the calculations real baud rates for the real length of the network between a node that transmits data and a node that receives data.

Related works
The CAN in Automation association on its official site can-cia.org written that the network CANopen has:
- baud rate is 1 Mbps at the distance of 25 meters;
- baud rate is 800 kbps at the distance of 50 meters;
- baud rate is 500 kbps at the distance of 100 meters;
- baud rate is 250 kbps at the distance of 250 meters;
- baud rate is 125 kbps at the distance of 500 meters;
- baud rate is 50 kbps at the distance of 1000 meters;
- baud rate is 20 kbps at the distance of 2500 meters;
- baud rate is 10 kbps at the distance of 5000 meters.

In view of the above, an important scientific and practical task is to develop a mathematical model for the dependence of baud rates of the length of the network CANopen. Such a mathematical model should be able to determine the baud rate at any point of the digital network CANopen. This will improve the design quality and allow the creation of control systems that meet the consumer's requirements without excessive cost overruns.

Development of a mathematical model of baud rate dependence from the length of the specialized digital CANopen network
In order to analyze the above data, these results were noted on a plane where the abscissa axis reflects the length of network in meters and the ordinate axis reflects the baud rate in kbps (Fig. 1).

![Fig. 9. Information about baud rate on the network CANopen on its segments of different length from the website of CAN in Automation Association](image1)

Initially, the search for a mathematical model as an exponential function was performed (Fig. 2).

![Fig. 10. Exponential model of baud rate dependence on CANopen network segment length](image2)
Figure 2 shows that the graph of the exponential model does not properly reflect the baud rate compared to the data obtained by the CAN in Automation Association for segments of length 25, 50, 100, 250, 500, 1000, 2500, 5000 m.

Then a search for a mathematical model as a logarithmic function was performed (Fig. 3).

![Figure 11. Logarithmic model of baud rate dependence on CANopen network segment length](image)

Figure 3 shows that the graph of the logarithmic model does not properly reflect the data rate compared to the data obtained by the CAN in Automation Association for segments 25, 50, 100, 250, 500, 1000, 2500, 5000 m in length.

Then a search for a mathematical model as a polynomial function was performed (Fig. 4).

![Figure 12. Polynomial model of baud rate dependence on CANopen network segment length](image)

It can be seen from Figure 4 that the graph of the polynomial model displayed on it does not properly reflect the baud rate compared to the data obtained by the CAN in Automation Association for segments 25, 50, 100, 250, 500, 1000, 2500, 5000 m in length.

Then a search for a mathematical model as a power function was performed (Fig. 5).

![Figure 13. Power model of the dependence of the baud rate on the CANopen network segment length](image)
Figure 5 shows that the graph of the power model does not properly reflect the baud rate compared to data obtained by the CAN in Automation Association for segments 25, 50, 100, 250, 500, 1000, 2500, 5000 m in length. However, this model is closest to that of the models considered. Therefore, it was decided to investigate the results of modeling the dependence of data rate on the length of the CANopen network segment using a power model:

$$v = 27572 \cdot x^{-0.908}$$  \hspace{1cm} (1)

The results of calculating the baud rates of model (1) for a given segment length of the CANopen network and the errors of such calculations are shown in Table 1.

<table>
<thead>
<tr>
<th>Segment length, m</th>
<th>The baud rate is defined by CiA, kbps</th>
<th>The baud rate is calculated by model (1), kbps</th>
<th>Error in calculating baud rate using model (1), kbps</th>
<th>Error in calculating baud rate using model (1), %</th>
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<td>1482.992</td>
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<td>-48.299</td>
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<td>50</td>
<td>800</td>
<td>790.321</td>
<td>9.679</td>
<td>1.210</td>
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<td>250</td>
<td>183.290</td>
<td>66.710</td>
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<td>97.680</td>
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<td>52.056</td>
<td>-2.056</td>
<td>-4.111</td>
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<td>2500</td>
<td>20</td>
<td>22.654</td>
<td>-2.654</td>
<td>-13.268</td>
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<tr>
<td>5000</td>
<td>10</td>
<td>12.073</td>
<td>-2.073</td>
<td>-20.727</td>
</tr>
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</table>

Table 5

Results of modeling the dependence of baud rate on the length of the CANopen network segment using a power model

After that, the absolute values of the baud rate were calculated by model (1) for a given segment length of the CANopen network that were analyzed and the errors of such calculations (Table 1). It is established that:

- in the area from 1000 m to 5000 m (1000 m, 2500 m, 5000 m) with almost the same error (from -2.056 kbps to -2.654 kbps);
- from 100 to 250 m (100 m, 250 m) with similar errors (78,819 kbps and 66,710 kbps);
- the segment with a length of 25 m is the largest error (-482,992 kbps) and has the opposite sign to the one that has the error at the next point (50 m).

The above facts should be taken into account when selecting the CANopen network segments for which mathematical models will be developed as to how the baud rate depends on the length of the CANopen network segment.

The next step was to define the boundaries of each of the simulation areas.

Analyzing the above and Table 1, it was decided that one model would be from 1000 m to 5000 m.

The analysis of the power model of the dependence of the baud rate on the CANopen network segment length (Fig. 5) and the results of the modeling of the baud rate dependence of the CANopen network segment length using the power model gave sufficient reason to believe that in the CANopen network segment from 1000 m to 5000 m to look for a mathematical model of the transmission rate dependence of the CANopen network segment length as a power function. As a result of the conducted researches it is established that in the CANopen network area from 1000 m to 5000 m, the dependence of the baud rate on the segment length can be described by the dependence:

$$v = 50000 \cdot x^{-1}$$  \hspace{1cm} (2)

The results of calculating the baud rates of model (2) for the segment length from 1000 m to 5000 m CANopen network and the errors of such calculations are shown in Table 2.

<table>
<thead>
<tr>
<th>Segment length, m</th>
<th>The baud rate is defined by CiA, kbps</th>
<th>The baud rate is calculated by model (2), kbps</th>
<th>Error in calculating baud rate using model (2), kbps</th>
<th>Error in calculating baud rate using model (2), %</th>
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<td>1000</td>
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<td>2500</td>
<td>20</td>
<td>20.000</td>
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<td>5000</td>
<td>10</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
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Table 6

Results of calculation of baud rates of model (2) for segment length from 1000 m to 5000 m of CANopen network and errors of such calculations

The power model of the dependence of the baud rate on the CANopen segment length (1000 m to 5000 m) is shown in Fig. 6.
Figure 6 and table 2 show that the graph of the power model displayed on it reflects the change in baud rate depending on the length of the CANopen network segment (in the range from 1000 m to 5000 m) and the values of the baud rate obtained by CiA for 1000 m network sections, 2500 m and 5000 m correspond to them.

After that, a mathematical model of the dependence of the baud rate on the segment length of the CANopen network in the range from 25 to 100 m was searched.

As a result of the conducted research it is established that in the CANopen network area from 25 m to 100 m, the dependence of the baud rate on the segment length can be described by the dependence:

$$v = 0.0267x^2 - 10x + 1233.3$$  \hspace{1cm} (3)

The results of calculating the baud rates of model (3) for the segment length from 25 m to 100 m CANopen network and the errors of such calculations are shown in table 3.

<table>
<thead>
<tr>
<th>Segment length, m</th>
<th>The baud rate is defined by CiA, kbps</th>
<th>The baud rate is calculated by model (3), kbps</th>
<th>Error in calculating baud rate using model (3), kbps</th>
<th>Error in calculating baud rate using model (3), %</th>
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<td>25</td>
<td>1000</td>
<td>999.988</td>
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<td>100</td>
<td>500</td>
<td>500.300</td>
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The polynomial model of the dependence of the baud rate on the CANopen network segment length (25 m to 100 m) is shown in Fig. 7.
Figure 7 shows that the graph of the polynomial model displayed on it shows the change in baud rate depending on the length of the CANopen network segment (in the range from 25 m to 100 m) and the values of the baud rate obtained by CiA for sections of network length of 25 m, 50 m and 100 m correspond to them with an error of less than 0.1% (from 0.001% to 0.060%).

After that, a mathematical model of the baud rate dependence of the CANopen network segment length from 100 to 1000 m was searched.

The exponential model of the dependence of the baud rate on the CANopen segment length (100 m to 1000 m) is shown in Fig. 8.

![Fig. 8. Exponential model of baud rate dependence on CANopen segment length (100 m to 1000 m)](image)

The logarithmic model of the baud rate dependence on the CANopen network segment length (100 m to 1000 m) is shown in Fig. 9.

![Fig. 9. Logarithmic model of baud rate dependence on CANopen segment length (100 m to 1000 m)](image)

The polynomial model of the dependence of the baud rate on the CANopen segment length (100 m to 1000 m) is shown in Fig. 10.
The power model of dependence of baud rate on CANopen segment length (from 100 m to 1000 m) is shown in Fig. 11.

As a result of the conducted researches it is found that in the CANopen network area from 100 m to 1000 m, the dependence of baud rate on the segment length cannot be accurately described with one dependence. This is illustrated by figures 8-11. The most accurate of the mathematical models studied in the area from 100 to 1000 m is the power model:

$$v = 53184 \cdot x^{-0.991}$$  \hspace{1cm} (4)

However, it also has quite high modeling errors from 10% to 13% (Table 4).

Table 8

<table>
<thead>
<tr>
<th>Segment length, m</th>
<th>The baud rate is defined by CiA, kbp/s</th>
<th>The baud rate is calculated by model (1), kbp/s</th>
<th>Error in calculating baud rate using model (4), kbp/s</th>
<th>Error in calculating baud rate using model (4)</th>
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<td>554.346</td>
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<td>250</td>
<td>250</td>
<td>223.575</td>
<td>26.425</td>
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<td>112.487</td>
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<td>1000</td>
<td>50</td>
<td>56.595</td>
<td>-6.595</td>
<td>-13.191</td>
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</table>

In view of the above, it was decided to split the section from 100 m to 1000 m into three segments and to build separate models for each of them.
Initially, the segment from 250 m to 500 m was investigated.
It should be noted that the most relevant general model of the baud rate dependence of the CANopen segment length (1) depicted in Figure 5 is the power model. For segments from 1000 m to 5000 m, the model of the baud rate dependence on the CANopen segment segment length (2) shown in Figure 6 is also the power model. Therefore, it was decided to search for a mathematical model in the segment from 250 m to 500 m as a power model. As a result of the conducted researches it is established, mathematical dependence:

\[ v = 62500 \cdot x^{-1} \]  

(5)  

The power model of dependence of baud rate on the CANopen segment segment length (from 250 m to 500 m) is shown in Fig. 12.

![Power model of baud rate dependence of CANopen network segment length (250 m to 500 m)](image)  

Fig. 20. Power model of baud rate dependence of CANopen network segment length (250 m to 500 m)

The power model of dependence of baud rate on CANopen segment length (from 500 m to 1000 m) is shown in Fig. 13.

![Power model of baud rate dependence of CANopen segment length (500 m to 1000 m)](image)  

Fig. 21. Power model of baud rate dependence on CANopen segment length (500 m to 1000 m)

The power model of dependence of baud rate on CANopen segment length (from 100 m to 250 m) is shown in Fig. 14.

![Power model of baud rate dependence of CANopen segment length (100 m to 250 m)](image)  

Fig. 22. Power model of baud rate dependence of CANopen segment length (100 m to 250 m)
Thus, the general mathematical model of the transmission rate dependence on the length of the CANopen network segment will be:

\[
\begin{align*}
v &= 0.0267x^2 - 10x + 1233.3, \quad 25 \leq x \leq 100 \\
v &= 16290 \cdot x^{-0.756}, \quad 100 < x \leq 250 \\
v &= 62500 \cdot x^{-1.322}, \quad 500 < x \leq 1000 \\
v &= 50000 \cdot x^{-1}, \quad 1000 < x \leq 5000 \\
\end{align*}
\]  
(6)

The conducted researches give the grounds to assert that in general the whole range of distances can be divided into two parts:

- from 25 m to 100 m, where the dependence of the baud rate on the length of the CANopen network segment is described by polynomial dependence 3;
- from 100 m to 5000 m, baud rates from the length of the CANopen network segment is described by a power model 4 or a set of power models.

The developed generalized model 6 allows to determine the baud rate at any distance between the source of transmission and the recipient of information.

Conclusions

The transmission is one of the most important in the design of specialized computer networks, and for the CANopen network it is known only at a few points from 5000 m. When designing computer systems for the control and management of industrial objects that require a guaranteed response to random events in a fixed period of time, developers, as a rule, in cases where the length of the segment is in the intermediate sections between the specified points, calculate the speed value for a longer segment length than is real. Considering that there can be tens of thousands of nodes in complex objects, the sum deviates significantly from the real performance indicators in the direction of increasing the costs of unnecessary increase in system performance due to the inability to use in the calculations of real baud rates for the real segment length between a node that transmits data and receives data.

A mathematical model for the dependence of the baud rate of the length of the CANopen-network has been developed. Such a mathematical model should be able to determine the baud rate at any point in the segment of the dedicated CANopen digital network. This will improve the design quality and allow the creation of control systems that meet the requirements of the consumer without excessive cost overruns.

References

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<table>
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<th>Email</th>
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<th>Affiliation</th>
</tr>
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<tbody>
<tr>
<td>Serhii Babchuk</td>
<td>PhD, Associate Professor of Computer Systems &amp; Networks Department, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine, e-mail: <a href="mailto:serhii.babchuk@nung.edu.ua">serhii.babchuk@nung.edu.ua</a></td>
<td><a href="https://orcid.org/0000-0002-1746-5731">https://orcid.org/0000-0002-1746-5731</a></td>
<td>Кандидат технічних наук, доцент кафедри комп’ютерних систем та мереж, Івано-Франківський національний технічний університет нафти і газу, Івано-Франківськ, Україна.</td>
<td></td>
</tr>
<tr>
<td>Taras Humeniuk</td>
<td>PhD, Associate Professor of Computer Systems &amp; Networks Department, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine, e-mail: <a href="mailto:taras.humeniuk@nung.edu.ua">taras.humeniuk@nung.edu.ua</a></td>
<td><a href="https://orcid.org/0000-0003-2610-2550">https://orcid.org/0000-0003-2610-2550</a></td>
<td>Кандидат технічних наук, доцент кафедри комп’ютерних систем та мереж, Івано-Франківський національний технічний університет нафти і газу, Івано-Франківськ, Україна.</td>
<td></td>
</tr>
<tr>
<td>Iryna Babchuk</td>
<td>Student, Computer Systems &amp; Networks Department, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine, e-mail: <a href="mailto:ir618a@gmail.com">ir618a@gmail.com</a></td>
<td></td>
<td>Студентка кафедри комп’ютерних систем та мереж, Івано-Франківський національний технічний університет нафти і газу, Івано-Франківськ, Україна.</td>
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