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# USE OF ANALYTICAL MODEL FOR SYNTHESIS OF ALGORITHMS FOR CONTROL OF TRANSPORT CONVEYOR PARAMETERS

*This study presents a methodology for synthesizing optimal control algorithms for the flow parameters of a conveyor-type transport system with a variable transport delay. A multi-section transport conveyor is a complex dynamic system with a variable transport delay. The transport conveyor is an important element of the production system, used to synchronize technological operations and move material. The Analytical PiKh-model of the conveyor section was used as a model for designing a control system for flow parameters. The characteristic dimensionless parameters of the conveyor section are introduced and the similarity criteria for the conveyor sections are determined. The model of a conveyor section in a dimensionless form is used to develop a methodology for synthesizing algorithms for optimal control of the flow parameters of a transport conveyor section. The dependencies between the value of the input and output material flow of the section are determined, taking into account the initial distribution of the material along the conveyor section, variable transport delay, restrictions on the specific density of the material, and restrictions on the speed of the belt. The dependencies between the value of the input and output material flow for the case of a constant transport delay are analyzed. A technique for synthesizing algorithms for optimal belt speed control based on the PiKh-model of a conveyor section is presented. As a simplification, a two-stage belt speed control is considered. Particular attention is paid to the methodology for synthesizing optimal control algorithms based on the energy management methodology (TOU-Tariffs). The criteria of control quality are introduced and problems of optimal control of flow parameters of the transport system are formulated. Taking into account differential connections and restrictions on phase variables and admissible controls, which are typical for the conveyor section, the Pontryagin function and the adjoint system of equations are written. As examples demonstrating the design of optimal control, algorithms for optimal control of the flow parameters of the transport system are synthesized and analysis of optimal controls is performed.*
*Keywords: conveyor, distributed system, PDE-model, production line, belt speed*

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# ВИКОРИСТАННЯ АНАЛІТИЧНОЇ МОДЕЛІ ДЛЯ СИНТЕЗУ АЛГОРИТМІВ КЕРУВАННЯ ПАРАМЕТРАМИ ТРАНСПОРТНОГО КОНВЕЄРА

*Дане дослідження презентує методику синтезу алгоритмів оптимального керування потоковими параметрами транспортної системи конвеєрного типу із змінною транспортною затримкою. Багатосекційний транспортний конвеєр – це складна динамічна система із змінною транспортною затримкою. Транспортний конвеєр є важливим елементом виробничої системи, що використовується для синхронізації технологічних операцій та переміщення матеріалу. Як модель для проектування системи управління потоковими параметрами використана аналітична PiKh-модель секції конвеєра. Введено характерні безрозмірні параметри секції конвеєра та визначено критерії подібності до секцій конвеєра. Модель секції конвеєра у безрозмірному вигляді використана для розробки методики синтезу алгоритмів оптимального керування потоковими параметрами секції транспортного конвеєра. Визначено залежності між значенням вхідного та вихідного потоку матеріалу секції з урахуванням початкового розподілу матеріалу вздовж секції конвеєра, змінної транспортної затримки, обмежень на питому щільність матеріалу та обмежень на швидкість стрічки. Проаналізовано залежність між значенням вхідного та вихідного потоку матеріалу для випадку постійної транспортної затримки. Представлено методику синтезу алгоритмів оптимального управління швидкістю стрічки, засновану на PiKh-моделі секції конвеєра. Як спрощення розглянуто двоступінчасте регулювання швидкості стрічки. Особливу увагу приділено методиці синтезу алгоритмів оптимального управління, що ґрунтується на методології енергоменеджменту (TOU-Tariffs). Введено критерії якості керування та сформульовано завдання оптимального керування потоковими параметрами транспортної системи. З урахуванням диференціальних зв'язків та обмежень на фазові змінні та допустимі керування, характерні для конвеєрної ділянки, записана функція Понтрягіна та приєднана система рівнянь. Як приклади, що демонструють проектування оптимального керування, синтезовано алгоритми оптимального керування потоковими параметрами транспортної системи та проведено аналіз оптимальних керувань.*
*Ключові слова: конвеєр, розподілена система, PDE-модель, потокова лінія, швидкість стрічки*

## Introduction

Industry 4.0 is the next step in the industrial revolution. Industry 4.0 includes the following requirements for modern production: completion of full automation of digital production; control of technological parameters in real-time; use of intelligent control systems with access to the global Internet; constant interaction with external production and marketing environments. Modern industrial technologies in process control are forced to use artificial intelligence and additive forecasting methods. At the same time, operational control of production parameters using sensors and sensors is a separate problem. Particular importance is given to this problem in enterprises with a continuous production method, in which the main element of material transportation, as a rule, is a conveyor [1]. The use of modern transport systems in the production process can significantly increase the efficiency of algorithms for the operational control of the flow parameters of the production system. The conveyor acts as a link connecting production modules and uses intelligent control components and various industrial sensors to improve the efficiency of the production process, which determine the state of the parameters of the transport system [2].

When considering the concept of "Industry 4.0", conveyor transport is important in the mining industry [3]. This is determined by the following circumstances: a) firstly, due to the low unit cost, the transport conveyor is the most suitable means for transporting the material [4, 5]; b) secondly, modern transport systems consist of a large number of sections, each of which operates with certain flow parameters [6]. These parameters for sections of the conveyor system are interconnected, and contain a variable transport delay; c) thirdly, the length of the transportation route for a separate section reaches tens of kilometers and the trend towards an increase in the length of sections continues to persist [7]; e) fourthly, the specific cost of transporting material for the standard mode of operation of the transport system is 20% of the total cost of extracting the material [8]. With an increase in the number of sections and the length of the route of the transport system, the cost of transporting the material increases non-linearly and can make up the bulk of the cost of extracting the material; f) fifthly, under the standard mode of operation, the material loading factor of the conveyor section is 0.5–0.7 of the full conveyor load. Increasing the filling factor of the conveyor section with material allows for reducing transportation costs by 30-50%. However, quite often the transport system operates with a fill factor below the norm due to the use of efficient real-time control systems; f) sixthly, restrictions are imposed on the operation modes of the conveyor section, associated, for example, with the propagation of long-wave disturbances in the conveyor belt, which can cause the destruction of the conveyor belt.

## Related Works

For conveyor systems, the cost of transportation is directly related to the cost of electricity that is consumed to transport the material. One of the main methods of reducing energy consumption, which is widely used in the transport of material, involves the use of conveyor belt speed control systems [10]. This method consists in increasing the linear density of the material at the input of the conveyor section by reducing the speed of the belt, which implies an increase in the material loading factor of the conveyor section. The next most effective method for reducing energy consumption is based on the use of a material flow control system from the input bunker[11]. In this case, an increase in the linear density of the material at the input of the conveyor section is achieved by increasing the input flow of material from the accumulation bunker of the conveyor section. The third method for reducing the specific energy consumption for material transportation is based on the use of energy management methodology [12]. The method is based on the use of transport system mode control systems. The operation of sections of the transport conveyor is carried out at times when the price of electricity is low. And the last significant method for reducing specific energy consumption is the method using reverse conveyors [6]. Reducing the specific cost of energy consumption occurs due to a change in the structure of the transport route as a result of a change in the direction of the flow of material in the supply section of the conveyor to the opposite direction. The control system determines the optimal route for the movement of material from several possible ones. For the synthesis of optimal control algorithms, a set of models for the section of the transport conveyor is used. The most common models of a conveyor section are FEM models of a transport system based on the finite element method [13] and DEM models of a transport system using the discrete element method [14]. These models belong to the class of numerical models and are used in flow parameter control systems that require taking into account the non-uniformity of the material flow and the variable transport delay when the material moves along the transport route. These models require significant computational resources, which imposes significant restrictions on their use in the synthesis of algorithms for optimal control of the flow parameters of the transport system. The transition from the model of a single-section transport system to the model of a multi-section transport system, consisting of several dozen sections [6], becomes a practically unsolvable problem. This is due to the almost absence of papers devoted to the design of control systems for the flow parameters of the transport conveyor, represented by numerical models. As an alternative to the class of numerical models for describing multi-section transport systems, a separate class can be proposed, which is based on models of regression equations [16] or a neural network [17]. However, to build optimal control systems using models based on regression equations or using a neural network, a sufficiently large amount of test data is required, which is not available for non-stationary modes of operation of the transport system. This circumstance is a strong limitation for the synthesis of control systems using an alternative class of models. Thus, the above analysis clearly identified the problems that make it difficult to synthesize algorithms for optimal control of the flow parameters of a modern transport conveyor: a) taking into account the uneven flow of material along the transportation route in the presence of a variable transport delay; b) a trend towards an increase in the number of sections of the transport system. The solution to these problems requires the improvement of existing models and algorithms for optimal control of the flow parameters of the transport conveyor.

## Purpose

In connection with the above circumstances, taking into account the fact that the industrial Internet of things provides an opportunity to create fully automated conveyor systems, the synthesis of algorithms for optimal control of the transport system flow parameters requires not only the improvement of existing and the creation of new models of the transport conveyor, which allow us to consider the transport conveyor as an object of intelligent control but also the improvement of methods for applying models for the synthesis of algorithms for optimal control of the flow parameters of the transport conveyor. This study is devoted to demonstrating the methodology for building transport conveyor control systems based on the use of an analytical PiKh-model of the conveyor section.

**Description model of a separate section of the conveyor line**

The conveyor line is a kind of production line, in a one-time approximation it has the form [1]

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=0, \qquad [\chi]_1(t,S)=[\chi]_{1\Psi}(t,S) \tag{1}$$

under initial and boundary conditions

$$[\chi]_0(0,S)=\Psi(S) \qquad [\chi]_1(t,0)=\lambda(t) \tag{2}$$

where $S_d$ – the length of the section in the phase space [18]); $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ is the distribution density and the material flow at the moment of time $t$ at the position, characterized by the coordinate $S\in[0,S_d]$; $\Psi(S)$ is initial distribution along the transport route; $[\chi]_{1\Psi}(t,S)$ is the normative material flow along a transport route; $\lambda(t)$ is the input material flow.

The fact that materials move at a speed that is independent of their location on the section belt allows the material flow $[\chi]_1(t,S)$ to be expressed in terms of their density $[\chi]_0(t,S)$ and the speed $a=a(t)$ of the belt. The speed of the belt can be constant or have a continuous or step regulation in time depending on the load. This made it possible to write the closed system of equations (1)–(2) in the following form [19]:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=\delta(S)\lambda(t), \qquad [\chi]_1(t,S)=a(t)\cdot[\chi]_0(t,S), \qquad \int_{-\infty}^{\infty}\delta(S)dS=1, \tag{3}$$

$$[\chi]_0(0,S)=\mathrm{H}(S)\Psi(S), \qquad \mathrm{H}(S)=\begin{cases}0, & S<0,\\ 1, & S\geq 0,\end{cases} \qquad S\in[0;S_d]. \tag{4}$$

The system of equations (3)–(4) models the flow of material moving along the transportation route. The right side of equation (3) $\delta(S)\lambda(t)$ takes into account the source of the material with the intensity characterizing the line capacity $\lambda(t)$, characterizing the line capacity (ton/hour). At the initial moment of time $t=0$ there is material on the conveyor line, which is distributed along the transportation route with linear density $[\chi]_0(0,S)$. The function $\delta(S)$ determines the place where the material enters the conveyor line: $S=0$. The system of equations is closed with respect to flow parameters $[\chi]_0(t,S)$ и $[\chi]_1(t,S)$. The closure condition shows the independence of the belt speed from the place of transportation and allows you to construct a solution to the system of equations (3)–(4) with respect to the flow parameters $[\chi]_0(t,S)$ and $[\chi]_1(t,S)$. A schematic diagram of a conveyor section with an accumulating bunker at the input is shown in Fig. 1 [20]. The material flow must be supplied to the input of the conveyor line from the bunker with the intensity necessary to ensure the required specified material flow at the output.

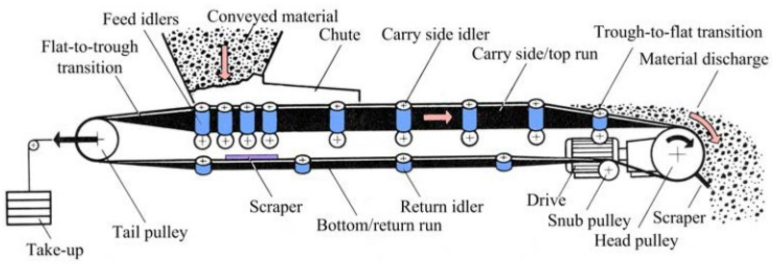


**Fig.1. Schematic diagram of the conveyor line [20]**

Let us supplement the system of equations (3)–(4) with an equation that determines the state of the bunker:

$$\frac{dN(t)}{dt}=\lambda_{in}(t)-\lambda(t), \qquad N(0)=N_0, \qquad 0\leq N\leq N_{\max}, \quad 0\leq\lambda(t)\leq\lambda_{\max}, \tag{5}$$

where $N(t)$ the current amount of materials in the bunker with a capacity of $N_{\max}$. The flow of materials to the input to the accumulation bunker is a given value $\lambda_{in}(t)$. Let us represent the system of equations (3)–(5) in a dimensionless form and will use the dimensionless parameters [21]:

$$\tau = \frac{t}{T_d}, \qquad \xi = \frac{S}{S_d}, \tag{6}$$

$$\theta_0(\tau,\xi) = \frac{[\chi]_0(t,S)}{\Theta}, \quad \psi(\xi) = \frac{\Psi(S)}{\Theta}, \quad n(\tau) = \frac{\mathrm{N(t)}}{S_d\Theta}, \quad \gamma(\tau) = \lambda(t)\frac{T_d}{S_d\Theta}, \quad \gamma_{in}(\tau) = \lambda_{in}(t)\frac{T_d}{S_d\Theta}, \tag{7}$$

$$g(\tau) = a(t)\frac{T_d}{S_d}, \ \Theta = \max\left\{\Psi(S), \frac{\lambda(t)}{a(t)}\right\}, \delta(\xi) = S_d\delta(S), \ H(S_d\xi) = H(S), \qquad a(t) \neq 0. \tag{8}$$

The value of the specific load on the conveyor belt should not exceed the maximum permissible value

$$\frac{\lambda(t)}{a(t)} = [\chi]_0(t,0) \leq [\chi]_{0\max}.$$

With the dimensionless value $n(\tau) = 1.0$ and $\Theta = [\chi]_{0\max}$ the accumulating bunker contains the amount of material $\mathrm{N(t)} = S_d\Theta$, that will fill the conveyor line along the entire length with the maximum allowable rock distribution density $[\chi]_0(t,S) = [\chi]_{0\max}$. Taking into account the notation (6)–(8), the balance equation for the flow parameters of the conveyor line is written in the dimensionless form [21]:

$$\frac{\partial\theta_0(\tau,\xi)}{\partial\tau} + g(\tau)\frac{\partial\theta_0(\tau,\xi)}{\partial\xi} = \delta(\xi)\gamma(\tau), \qquad \theta_0(0,\xi) = \mathrm{H}(\xi)\psi(\xi), \tag{9}$$

$$\frac{dn(t)}{dt} = \gamma_{in}(\tau) - \gamma(\tau), \quad n(0) = n_0, \quad 0 \leq n(\tau) \leq n_{\max}, \quad 0 \leq \gamma(\tau) \leq \gamma_{\max}. \tag{10}$$

The solution of the system of equations (9) has the form:

$$\theta_0(\tau,\xi) = \left[H(\xi) - H\left(-G(\tau_\xi)\right)\right]\frac{\gamma(\tau_\xi)}{g(\tau_\xi)} + H\left(-G(\tau_\xi)\right)\psi\left(-G(\tau_\xi)\right), \tag{11}$$

$$G(\tau) = \int g(\tau)d\tau, \qquad \tau_\xi = G^{-1}\left(G(\tau) - \xi\right). \tag{12}$$

For a constant speed of the conveyor belt $g(\tau) = g_0 = const$ expressions (12) can be represented as

$$G(\tau) = g_0\tau, \qquad \tau_\xi = \frac{G(\tau) - \xi}{g_0} = \frac{g_0\tau - \xi}{g_0} = \tau - \frac{\xi}{g_0}, \tag{13}$$

whence the expression for the distribution material density along the transportation route at an arbitrary point in time $\theta_0(\tau,\xi)$ along the transportation route at an arbitrary point in time $\tau$

$$\theta_0(\tau,\xi) = \left[H(\xi) - H\left(\xi - g_0\tau\right)\right]\frac{\gamma\left(\tau - \frac{\xi}{g_0}\right)}{g_0} + H\left(\xi - g_0\tau\right)\psi\left(\xi - g_0\tau\right). \tag{14}$$

For the transportation process by $\tau > \xi/g_0$

$$\theta_0(\tau,\xi) = \frac{\gamma\left(\tau - \frac{\xi}{g_0}\right)}{g_0}, \qquad \theta_1(\tau,\xi) = \gamma\left(\tau - \frac{\xi}{g_0}\right), \qquad \tau > \frac{\xi}{g_0}. \tag{15}$$

In order to determine the value of linear density $\theta_0(\tau,\xi)$ or material flow $\theta_1(\tau,\xi)$ at an arbitrary point $\xi$ at a point in time $\tau$, it is required to know the value of the input material flow to the conveyor line at a point in time

$\tau_\xi = \tau - \xi / g_0$. The material flow at the output from the conveyor line $\theta_1(\tau,1)$ at a constant belt speed is determined by the product $g_0$ и $\theta_0(\tau,1)$

$$\theta_1(\tau,1) = g_0 \theta_0(\tau,1) = g_0 \frac{\gamma(\tau - 1/g_0)}{g_0} = \gamma(\tau - 1/g_0). \tag{16}$$

In general, when the belt speed and the input material flow are variable, the expression for the material flow at the exit of the conveyor section is

$$\theta_1(\tau,1) = g(\tau)[H(1) - H(-G(\tau_1))]\frac{\gamma(\tau_1)}{g(\tau_1)} + H(-G(\tau_{1\xi}))\psi(-G(\tau_1)), \qquad \tau_1 = G^{-1}(G(\tau) - 1). \tag{17}$$

Expressions (16), (17) determine the value of the output material flow from the conveyor section, which can be used to design a control system for the flow parameters of a conveyor-type transport system.

**Results**

**1. The problem of optimal control of the conveyor belt speed**

Let us formulate the problem of constructing an optimal program for controlling the speed of a conveyor belt for a steady state operation $G(\tau) - 1 > 0$ of a conveyor line: determine the material output $\theta_1(\tau,1)$ from the conveyor line during a period of time $\tau = [0, \tau_k]$ with step control of the speed of the conveyor belt $u(\tau) = (u_1, u_2)$, $0 < u_1 < u_2 < \infty$, $u_1 = const$ , $u_2 = const$ , which leads to a minimum functionality

$$\int_0^{\tau_k} |\theta_1(\tau,1) - \vartheta(\tau)| d\tau \to \min \tag{18}$$

with differential connections (9)

$$\frac{\partial \theta_0(\tau,\xi)}{\partial \tau} + u(\tau)\frac{\partial \theta_0(\tau,\xi)}{\partial \xi} = \delta(\xi)\gamma(\tau), \qquad g(\tau) = u(\tau) \tag{19}$$

and restrictions

$$\theta_0(\tau,\xi) \geq 0, \tag{20}$$

and initial conditions

$$\theta_0(0,\xi) = \mathrm{H}(\xi)\psi(\xi). \tag{21}$$

Let us assume that prior to the introduction of control, the conveyor operated in a steady state at a constant speed $g(\tau)\big|_{\tau<0} = u_1$. The Pontryagin function and the adjoint system have the form:

$$H = -\left|\frac{\gamma(\tau - \Delta\tau_1)}{u(\tau - \Delta\tau_1)} u(\tau) - \vartheta(\tau)\right| + \psi_1 u(\tau) \to \max, \qquad \int_{\tau - \Delta\tau_1}^{\tau} u(\tau) d\tau = 1, \tag{22}$$

$$\frac{d\psi_1}{d\tau} = \frac{\partial H}{\partial \xi} = 0. \tag{23}$$

Since the right end of the phase trajectory is free, then $\psi_1(\tau_k) = 0$ and, therefore, $\psi_1(\tau) \equiv 0$, which allows us to write the Pontryagin function in the following form

$$H = -\left|\frac{\gamma(\tau - \Delta\tau_1)}{u(\tau - \Delta\tau_1)} u(\tau) - \vartheta(\tau)\right| \to \max. \tag{24}$$

Let us construct the optimal control of the speed of the conveyor belt for the case of supplying material to the input of the conveyor with a constant intensity $\gamma(\tau)=1$ with the existing demand, which is determined by a periodic function $\vartheta(\tau)=1+\sin(\omega\tau)$ [22]. Let's believe that the conveyor line is capable of operating in one of the speed modes: with the speed of the belt $u_1$ or $u_2$, $u(\tau)=(u_1,u_2)$. The calculation results are shown in Fig. 2 for different values of the stepwise control of the belt speed $u_1$ or $u_2$: $u(\tau)=(0.5, 2.0)$, Fig.2a; $u(\tau)=(0.8, 2.0)$, Fig.2b; $u(\tau)=(1.0, 2.0)$, Fig.2c; $u(\tau)=(1.2, 2.0)$, Fig.2d; $u(\tau)=(1.8, 2.0)$, Fig.2e. The maximization of function (22) determines such a control $u(\tau)$, which the output of products on the conveyor line is ensured with a minimum deviation from the existing demand $\vartheta(\tau)$. Figure 2 shows the calculation of the optimal control $u(\tau)$ of the speed of the conveyor belt, depending on the demand $\vartheta(\tau)$. Graphs (a–e) on the left side in Fig. 2 represent the dependence of optimal control $u(\tau)$ on the value of demand $\vartheta(\tau)$. On the right, for each variant of the dependence $u(\tau)$, $\vartheta(\tau)$ a graph of the family of characteristics with breaks at the time of switching the speed mode is presented. Each control mode (a–e), shown in Fig. 2, corresponds to the time dependence of the output flow $\theta_1(\tau,1)$ from the conveyor against the background of existing demand $\vartheta(\tau)$. For the step control speed option $u(\tau)=(0.5, 2.0)$, Fig. 2a, there are no conveyor belt speed switching modes.

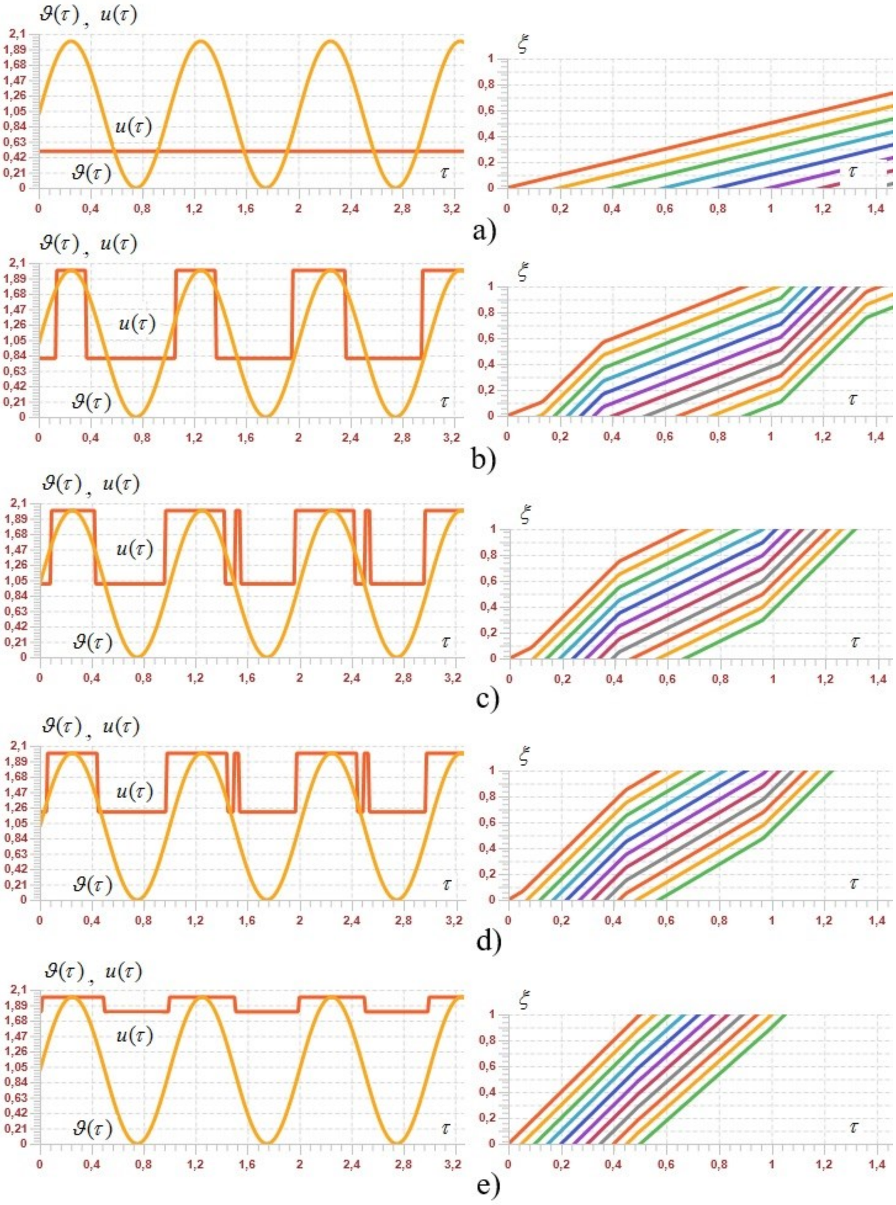


**Fig.2. Optimal control and a family of characteristics for operating modes**

The output material flow from the conveyor line $\theta_1(\tau,1)$ is constant and does not depend on the existing demand. This behavior is due, to some extent, to the fact that the initial speed of the conveyor line without control is taken as the smaller of the two, $g(\tau)|_{\tau<0} = u_1$, and a significant spread between the step value of the control speeds, which is characterized by a coefficient (Fig. 2a) $k_u = u_2 / u_1 = 4.0$. When decreasing $k_u$ by increasing the value $u_1$ the conveyor starts to operate in a two-speed mode (Fig. 2b), and the family of characteristics at these switching moments has kinks At the same time, the output flow from the section $\theta_1(\tau,1)$ is determined by the behavior of demand $\vartheta(\tau)$ and takes one of three values over time {0.4; 1.0; 2.5}, which is determined by the relation $\frac{u(\tau)}{u(\tau-\Delta\tau_1)} = \left\{\frac{2.0}{0.8}, \frac{0.8}{0.8}, \frac{0.8}{2.0}, \frac{2.0}{2.0}\right\}$.

The unevenness of material density $\theta_0(\tau,\xi)$ is set by the speed mode of the conveyor line and takes one of two values $\theta_0(\tau,\xi) = \frac{\gamma(\tau_\xi)}{g(\tau_\xi)} = \left\{\frac{1.0}{0.8}, \frac{1.0}{2.0}\right\}$. The duration of the operation of the section in the control mode $u(\tau) = u_2$ increases with each switch and reaches the steady state (Fig. 2b). With a subsequent decrease, short-term peak switchings are added (Fig. 2c, Fig. 2d), which disappear with a further decrease $k_u$ (Fig. 2e).

This behavior of the control function is explained by the fact that the speed control $u(\tau)$ of the section depends on the accepted control $u(\tau-\Delta\tau_1)$, where is $\Delta\tau_1$ a time-dependent value estimated by relation (22).

### 2. Synthesis of an algorithm for controlling the speed belt based on TOU-tariffs

The problem of constructing an optimal program for controlling the belt speed is formulated as follows: determine the modes of switching the belt speed during a period of time $\tau = [0, \tau_{24}]$ with the value of the price coefficients of the cost of electricity $z(\tau)$ (Fig. 3) with stepwise control of the belt speed $g(\tau) = u(\tau) = (u_1, u_2)$, $0 < u_1 < u_2 < \infty$, $u_1 = const$, $u_2 = const$, which leads to a minimum of the functional:

$$\int_0^{\tau_{24}} z(\tau)u(\tau)m(\tau)d\tau \to \min, \tag{25}$$

with differential connections

$$\frac{dm(\tau)}{d\tau} = \gamma_1(\tau) - \theta_1(1,\tau) = \gamma_1(\tau) - \gamma_1(\tau-\Delta\tau_1)\frac{u(\tau)}{u(\tau-\Delta\tau_1)}, \tag{26}$$

$$m(0) = 2(\theta_{0R} + \theta_{0C}) + \int_0^1 \psi(\xi)d\xi, \tag{27}$$

and a limit on the total amount of energy consumed per day

$$\int_0^{\tau_{24}} u(\tau)m(\tau)d\tau = b = const. \tag{28}$$

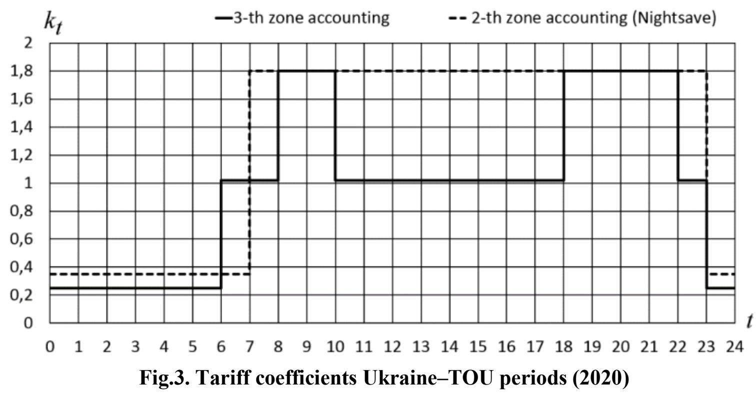


**Fig.3. Tariff coefficients Ukraine–TOU periods (2020)**

Equation (26) is written taking into account the dependence on the output material flow. The choice of a stepwise control mode is due to its prevalence in the control of transport systems [23, 24]. Equation (28) can be replaced by the differential equation

$$\frac{dx_b}{d\tau} = u(\tau)m(\tau), \qquad x_b(0) = 0, \qquad x_b(\tau_{24}) = b. \tag{28}$$

The Hamilton function and the adjoint system of equations for the problem under consideration has the form:

$$H = (\psi_b - z(\tau))u(\tau)m(\tau) + \psi_m\left(\gamma_1(\tau) - \gamma_1(\tau - \Delta\tau_1)\frac{u(\tau)}{u(\tau - \Delta\tau_1)}\right), \tag{29}$$

$$\frac{d\psi_b}{d\tau} = -\frac{\partial H}{\partial x_b} = 0, \tag{30}$$

$$\frac{d\psi_m}{d\tau} = (z(\tau) - \psi_b)u(\tau) \qquad \psi_m(\tau_{24}) = 0. \tag{31}$$

Equation (31) implies $\psi_b = C_b = const$. For the two-stage control mode $u(\tau) = (u_1, u_2)$ $0 < u_1 < u_2 < \infty$ the optimal belt speed corresponds to the maximum value of the Hamilton function (29). Switching points of control modes are determined by solving equations (26)-(31). To carry out quantitative calculations, let's take the value of the time parameter and the value of the sample $T_d = 1$ ( $hour$ ), $S_d = 20.5$ (km). The choice of the characteristic time value $T_d$ makes it possible to conveniently display the change in parameters during the day $\tau \in [0;24]$, and the choice of the characteristic length value corresponds to the consideration of extended transport conveyor (Sasol – Shondoni Overland (20.5 km single flight overland conveyor with multiple horizontal curves). Then the control modes $u(\tau) = (u_1, u_2)$, at speeds $a_1(\tau) = (1.0, 5.0)$ m/sec, will correspond to the dimensionless values of the belt speed $u(\tau) = (0.176,\ 0.878)$.

Let's consider the construction of a schedule for switching belt speed modes for tariff coefficients Ukraine–TOU periods (Fig. 3), when the intensity of material receipt is a constant value $\gamma_1(\tau) = 0.15$ with daily energy consumption $b = 6.5$. The selected value of the intensity of the incoming flow allows the transport system to operate in a two-speed mode $u(\tau) = (0.176,\ 0.878)$ (19). Increasing the belt speed $g(\tau)$ leads to an increase in the power consumption of the transport system $n_e(\tau)$. On the other hand, an increase in belt speed leads to a decrease in the linear density of the material $\theta_0(\tau, 0)$, entering the section input and, possibly, to a decrease in the mass $m(\tau)$. The next feature is that the conveyor belt is an accumulator of the material entering the section input. The limitation does not allow both the excess of the specific gravity and the overflow of the accumulator. The presence of upper and lower limits for the amount of material in the accumulator with a sufficiently long period of time of the transport system operation determines the ratio between the average intensity of the incoming flow and the average power consumption of the transport system.

An increase in the value of the intensity of the incoming material over a sufficiently long period of time of the operation of the transport system leads to an increase in the power consumption of the transport system.Эти особенности значительно усложняют синтез алгоритмов оптимального управления потоковыми параметрами транспортной системы. In this regard, to simplify the qualitative analysis, a constant value for the intensity of the incoming flow $\gamma_1(\tau)$ is taken. Belt speed control modes $u(\tau) = (u_1, u_2)$ for tariff coefficient values $k_\tau(\tau) = (0.35,\ 1.8)$ are shown in Fig. 4.

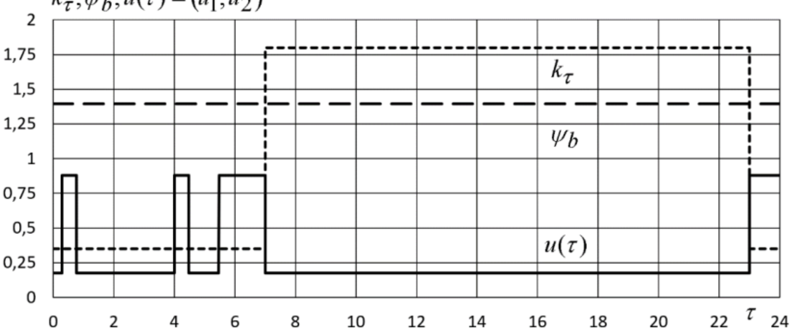

**Fig 4. Belt speed control modes for tariff coefficient values $k_\tau(\tau) = (0.35,\ 1.8)$ and $\psi(\xi) = 0.5 + 0.5\sin(2\pi\xi)$**

The value of the tariff coefficient $k_\tau = 1.8$ corresponds to the minimum belt speed $u_1$. There are several speed-switching points for the tariff coefficien $k_\tau = 0.35$. Switching on the interval $\tau \in [0.0;1.0]$ is related to the type of function (21), which determines the initial distribution of material along the route. Switching for the interval $\tau \in [4.0;5.0]$ repeats the switching mode for the interval $\tau \in [0.0;1.0]$ with a time shift equal to the amount of transport delay.

For the moments of time when the value of the transport delay has high values, the mass of the belt with the transported load also has high values (Fig. 4). An increase in the transport delay leads to an increase in the mass of material on the belt.

## Conclusions

The article discusses a technique using the PiKh–model of a conveyor section for the synthesis of algorithms for optimal control of the belt speed for a distributed transport system of a conveyor type. A method for constructing an algorithm based on the Pontryagin maximum principle using an analytical PiKh–model for a conveyor section is proposed. The use of the Pontryagin maximum principle together with the analytical PiKh–model allows us to provide acceptable accuracy for calculating the switching points of speed modes. To carry out numerical experiments, software was used that makes it possible to synthesize algorithms for optimal discrete control of the belt speed for practical and theoretical purposes. The presented technique for synthesizing optimal control algorithms makes it possible to take into account the initial distribution of material along the transportation route when choosing control modes.

The main advantage of the presented technique is that the synthesis of optimal control algorithms takes into account the variable transport delay for the conveyor section.

The prospect for further research is the development of a methodology for synthesizing algorithms for optimal continuous control of the speed of a conveyor belt in the design of control systems for multi-section transport conveyors.

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