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NEURAL NETWORK DECISION SUPPORT SYSTEM FOR FORMULATING A RACING TEAM STRATEGY

The research is aimed at studying the features of strategy formulation in Formula One auto racing, the factors that influence this process, and identifying ways to increase the effectiveness of the strategy through the use of artificial intelligence methods. During Formula One races, teams face a large number of challenges related to various aspects, in particular tire wear and degradation. Teams deal with them through the implementation of the strategy – determining the correct pit stop moment and choosing the appropriate type of tires. Based on the determination of factors influencing the rate of tire wear and degradation, an original feature space was formed. Analysis of the strategies of race winners in previous seasons showed that the maximum number of pit stops was three. Using the principal component analysis, a study of the original feature space was conducted, the least relevant features were identified and subsequently removed from the original data set. To solve the problem, a system was proposed and built, consisting of four modules, each of which is a multilayer feedforward artificial neural network. Using the inequality proposed by Widrow and the sample-based estimation of the Lipschitz constant, the minimum required number of neurons of the hidden layers for each neural network module was determined. During the training process, their number was specified to achieve acceptable prediction results. AdaMax was used as an optimization algorithm, and the Huber loss function was chosen to calculate the error of the networks output. The mean squared error of the resulting system prediction on the test set was 0.1. The use of such system will reduce the decision-making time of teams when formulating a racing strategy, which in turn will contribute to achieving higher results in races.

Keywords: auto racing, strategy, principal component analysis, artificial neural network, neural network system.

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НЕЙРОМЕРЕЖЕВА СИСТЕМА ПІДТРИМКИ ПРИЙНЯТТЯ РІШЕНЬ ДЛЯ ФОРМУВАННЯ СТРАТЕГІЇ ПЕРЕГОНОВОЇ КОМАНДИ

Дослідження спрямовано на вивчення особливостей формування стратегії в автомобільних перегонів серії «Формули-1», факторів, які впливають на цей процес, та визначення шляхів підвищення ефективності стратегії за рахунок використання методів штучного інтелекту. Під час перегонів «Формули-1» команди стикаються з великою кількістю викликів, що стосуються різних аспектів, зокрема зносу та деградації шин. Команди борються з ними через реалізацію стратегії – визначення правильного моменту піт-стопу та вибору підходящого типу шин. Спираючись на визначені фактори впливу на темпи зносу та деградації шин, було сформовано вихідний простір ознак. Аналіз стратегій переможців перегонів в сезонах попередніх років показав, що максимальна кількість піт-стопів дорівнювала трьом. Використовуючи метод головних компонент було проведено аналіз вихідного простору ознак, було визначено найменш релевантні змінні, які в подальшому було вилучено з вихідного набору даних. Для вирішення поставленої задачі було запропоновано та побудовано систему, яка складається з чотирьох модулів, кожний з яких являє собою багатошарову штучну нейронну мережу прямого поширення. Використовуючи нерівність, запропоновану Уідроу, та вибіркову оцінку константи Ліпшиця, було визначено мінімально необхідне число нейронів прихованих шарів для кожного нейромережевого модулю. У процесі навчання їх кількість було уточнено для досягнення прийнятних результатів прогнозування. У якості алгоритму оптимізації було використано AdaMax, для обчислення помилки роботи мереж було обрано функцію втрат Хьюбера. Середньоквадратична похибка прогнозування отриманої системи на тестовому наборі склала 0,1. Використання такої системи дозволить командам скоротити час прийняття рішень під час формування перегонової стратегії, що у свою чергу сприятиме досягненню вищих результатів в перегонах.

Ключові слова: автомобільні перегони, стратегія, метод головних компонент, штучна нейронна мережа, нейромережева система.

Introduction

There are a large number of types of automobile competitions, which differ in the kinds of tracks, the design of the cars, the peculiarities of the driving technique, etc. Despite the differences, the fact that the final result is influenced by many technical factors undoubtedly unites them.

The fundamental unit of any automobile championship is the team, each member of which must be an exceptional professional. The drivers must be hardy and in excellent physical shape, they must also understand the principles of operation of the car they drive. Such cars are fundamentally different from those used on public roads. The teams try to involve highly qualified constructers in the process of developing and building the car.

However, a professional driver and a car built using advanced technologies may not be enough to obtain the desired results in competitions. One of the main roles is played by the race strategy which consists in determining the optimal technical parameters of the car. To make informed and objective decisions in the process of forming a strategy, it is necessary to analyze large amounts of data, which makes the process laborious for people. Therefore, in such conditions, it is extremely relevant to implement a mechanism that will increase the efficiency of decision-making. For this purpose, mathematical modeling methods are used, in particular, Monte Carlo methods [1]. However, they have a number of limitations: they require significant computational resources and may be inefficient in large systems [2].

In this regard, it is appropriate to use alternative approaches, in particular, machine learning methods, which have better generalization ability [3] and higher efficiency in tasks with a complex input data structure.

It should also be noted that the own research activities of Formula One teams have very proprietary nature due to the high commercialization of the industry. It is the latest technical developments and scientific solutions that allow teams to gain an advantage in the highly competitive Formula One environment. This can be attributed to the small number of open academic publications on this subject, as well as limited access to official data and scientific research. Therefore, a systematic study of the approaches of different teams to the formulating of a racing strategy and the factors influencing this process remains incomplete and requires further research. This specific aspect of the research object additionally determines the relevance of developing a generalized methodology for finding the optimal elements of the racing strategy.

The main scientific contribution of this study is the further development of decision-making methods when formulating the racing strategy through the use of artificial intelligence tools, in particular, artificial neural networks.

Related Works

Analysis of sports results, their prediction and development of possible tactics for the course of competitions are current subjects of research in the scientific community. For this purpose, machine learning methods, data mining and artificial neural networks are used [4–7].

Such studies are quite often conducted within team sports [8–10]. In addition, in sports where one team is represented by several individual athletes: horse racing [11] and car racing [12, 13].

In the paper [14], the automation of strategic decision-making in races is presented by developing a virtual strategy engineer based on two artificial neural networks. As the studied neural networks, the authors considered a multilayer feedforward neural network and a recurrent neural network. To determine the architecture of the studied networks, the authors used hyperparameter tuning. The results of evaluating the performance of the obtained networks by F1-score showed values of 0.59 and 0.77, respectively. It can be assumed that such results were obtained due to the incorrect choice of hyperparameters.

In [15], another approach is presented to determine the optimal racing strategy by predicting tire energy. In addition to the vanilla recurrent neural network, its variants are used to solve the problem: Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU), as well as a hybrid neural network for processing time series Temporal Fusion Transformer (TFT) and the XGBoost algorithm. For each track, a sequence of tire energy values is determined, i.e., how they are used during a full racing lap. This makes it possible to observe which parts of the track are more demanding on tires – where their wear and degradation rates will be higher. The knowledge obtained can then be used to determine the time of the pit stop and the type of tires that needs to be installed. The root mean square error (RMSE) was used to evaluate the quality of the forecast. On the tracks that were present in the training set, the RMSE value for different models did not exceed 5.8 on average. For tracks that were not present in the training set, the maximum RMSE value was 5.9.

In general, the use of artificial intelligence methods for formulating a strategy in car racing has demonstrated success, although the definition of the network architecture should be based on a well-founded analysis and research results.

Purpose

The aim of the article is to increase the efficiency of the racing strategy through the use of artificial intelligence methods.

Formation of the initial feature space

The highest class of international racing for single-seater racing cars is Formula One. It is held in 24 rounds, each of which takes place in different countries around the world. Each round brings its own challenges.

One of these challenges is the configuration of a track. The shape of the tracks is not subject to strict restrictions, which makes each of them unique. The configuration of some tracks is suitable for overtaking during the race, while others hinder the possibility of gaining an advantage directly on the track.

In addition to the corners and straights, in the configuration of any track, there are zones that allow the activation of the drag reduction system (DRS). It gives the car moving behind a speed advantage to overtake. DRS zones are pre-defined and usually located on straights, where overtaking opportunities are most probable [16].

Other significant challenges include tire wear and degradation. Wear is the gradual deterioration of a tire tread over time. Tire degradation is the deterioration of a tire's performance caused by excessive wear and temperature imbalances. These phenomena can lead to a decrease in the grip, stability, and speed of a racing car, which can jeopardize a team's chances of winning.

Factors that influence the rate of wear and degradation include [17]: the rubber compounds used to manufacture different types of tires, the ambient and track temperatures, the driver's driving style, and the conditions on the track.

Track conditions are a determining factor in how quickly tires wear out. Each track has a number of characteristics that determine these conditions. Among them, Pirelli, a tire supplier to Formula One teams, highlights [18]: traction, braking difficulty, lateral force, tire load, downforce, abrasiveness, adhesion, and asphalt evolution.

In order to prevent significant and premature degradation of the rubber, teams take various measures regarding tires: maintaining optimal tire pressure, choosing the appropriate rubber compound, and preheating the tires.

It is also important to set the maximum camber angle limit at the end of the straight, i.e. before entering the corner. During braking, the weight of the car is transferred from the rear to the front, which leads to compression of the suspension. As it is compressed, the camber of the wheels gradually increases, which significantly affects how the weight is transferred when passing corners. Uneven weight distribution can lead to skidding and loss of control in corners which in turn will increase the load on the tires and accelerate their degradation.

In addition to selecting the optimal car settings, the fight against excessive wear and degradation of tires includes finding the optimal driving style, analysis of telemetry data, and strategy.

Strategy in Formula One refers to the carefully planned and implemented approach that teams use to maximize their performance in a race. First and foremost, race strategy concerns pit stops and tire management. Team strategists are tasked with predicting the rate of degradation, the time required to stop, and determining which types of tires are appropriate for a given track.

According to the rules [19], at least one stop must be made during a race, and at least two different types of tires must be used. One set is used by the driver for the start, the others are used during pit stops. Thus, the types of tires that a team chooses during a race depend on each other – in addition to defining pit stops as dependent variables, they must be considered as part of the original feature space.

To determine the number of pit stops to be considered, data presented on the official website of Pirelli [20] was analyzed for the period from 2019 to 2023. This time period was chosen because in 2019, changes were made to the tires used: instead of seven different types, drivers were provided with only three (soft, medium and hard) per race [21].

As a result of the analysis, it was determined that in the absence of force majeure, the maximum number of pit stops is 3. Thus, the maximum possible number of sets of tires that a driver uses per race is 4 – one starting set and three installed during pit stops.

Table 1 presents the symbols of the features and their meanings.

Table 1

Features of the original space

Type	№ feature	Name of feature	Meaning
Independent variables	1	Laps	Number of racing laps
	2	Length	Length of one racing lap
	3	Num. Turns	Number of turns
	4	Num. DRS Zones	Number of DRS zones
	5	Season	Season
	6	Temp. Ambient	Air temperature in °C
	7	Traction	Traction (in turns)
	8	Track Evolution	Evolution of asphalt pavement
	9	Lateral	Lateral forces
	10	Asphalt Abrasion	Abrasiveness of asphalt pavement
	11	Bracking	Difficulty of braking
	12	Asphalt Grip	Asphalt adhesion
	13	Tyre Stress	Tire load
	14	Downforce	Downforce
	15	Min. Starting Pressure (Front)	Minimum permissible front tire pressure at the start of the race
	16	Min. Starting Pressure (Rear)	Minimum permissible rear tire pressure at the start of the race
	17	EOS Camber Limit (Front)	Front wheel camber limit at the end of the straight
	18	EOS Camber Limit (Rear)	Rear wheel camber limit at the end of the straight
Dependent variables	19	Start	Type of tires installed at the start of the race
	20	Pit 1	Type of tires installed during the first pit stop
	21	Pit 2	Type of tires installed during the second pit stop
	22	Pit 3	Type of tires installed during the third pit stop

Proposed methodology and description of experimental studies

Data preprocessing is an important part of working with data: it allows extracting valuable information, simplify its use and analysis.

Data collected from the real world has many problems. Their nature depends on many reasons, sometimes beyond human control. Data problems can be classified into three groups [22]: too much data, too little data and fractured data. By understanding the problems inherent in the data and their underlying causes, conclusions can be drawn about the techniques that are most suitable for their processing.

Preprocessing approaches can be divided into three groups [22]: data transformation, information gathering, and generation of new information.

Data mining and machine learning methods typically work with the assumption that the input data is suitable for use. Therefore, when key data characteristics remain unidentified, the results obtained may be incomplete [22]. To increase the reliability of subsequent analysis, information gathering techniques can be used, as they allow a deeper understanding of the structure, quality, and underlying patterns of the data set. One such method is principal component analysis (PCA).

The main purpose of identifying principal components is to select appropriate attributes for data analysis. Identifying principal components allows you to reduce the dimensionality of a data set containing a large number of interrelated variables while preserving as much of the variation as possible. This reduction is achieved by transforming the original data into a new set of variables that are ordered in such a way that the first few retain most of the variation [22].

Table 2 presents the parameters of the PCA model that was constructed.

Table 2

Parameters of the resulting PCA model

Component	R^2X	R^2X (Cumul.)	Eigenvalues	Q^2	Limit	Q^2 (Cumul.)
1	0.383	0.383	6.89	0.316	0.062	0.316
2	0.162	0.545	2.913	0.156	0.065	0.423
3	0.096	0.64	1.722	0.044	0.069	0.448
4	0.076	0.716	1.36	0.062	0.073	0.483

One of the significant advantages of the principal component analysis is the ability to identify outliers. The analysis revealed that there were no outliers in the original data set, but there were minor deviations.

The principal component analysis is also used to identify relevant attributes. In [23], a study was conducted on the separation of the least significant features in the original data set. Figure 1 shows the results of determining the power for each feature in the original space.

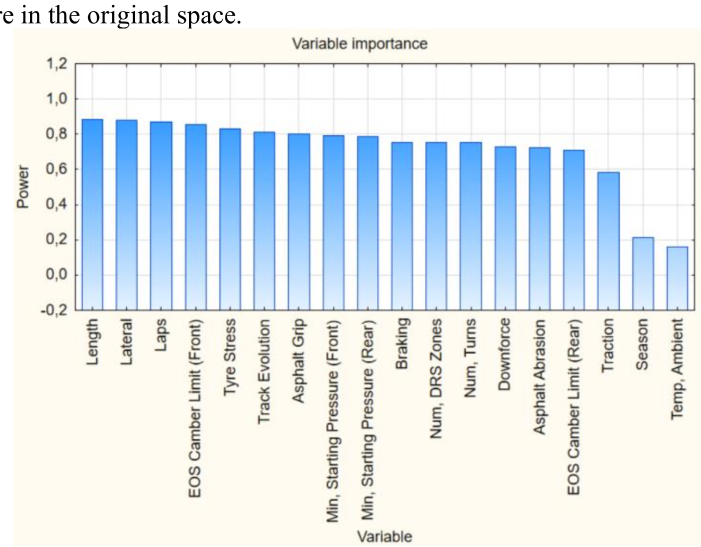

Fig. 1. Visualization of feature importance

It can be seen that the most significant features are Length and Lateral – their power values are close to 1. The least significant variables are Season and Temp. Ambient. Their power values are close to 0 (0.211 and 0.157).

As a result of the study, it was concluded that removing the least important features from the original data set will increase its statistical significance and improve the quality of processing by machine learning methods.

The presence of multiple outputs (the need to generate a sequence of tire sets that the driver should use) can have a negative impact on training. The need to predict multiple variables simultaneously can significantly slow down the training process, complicate the approximation problem, and require more computational effort to solve than for a single-output system. In addition, models with multiple outputs have a higher risk of overfitting, especially when the training sample contains a limited amount of data.

In addition to the disadvantages described above, the inexpediency of using a model with multiple outputs for strategy formation lies in the possible omission of the connection between the tire sets that will be recommended to be chosen.

Thus, based on the disadvantages of using networks with multiple outputs and taking into account the relationship between the variables that need to be predicted, in [24] it was proposed to use a system with the architecture presented in Figure 2. $X_1 - X_{16}$ – attributes from the original data set, $NM_1 - NM_4$ – neural network modules of the system, $y_1 - y_4$ – sets of tires.

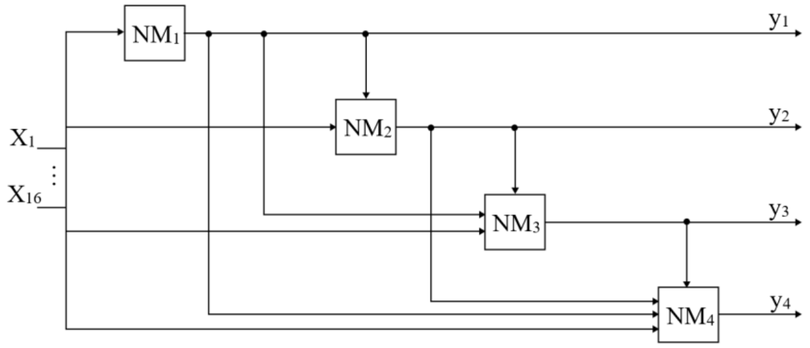

Fig. 2. Architecture diagram of the built system

As can be seen in the figure, each module of the system has one output, which determines the set of tires at each stage of the strategy. In addition, the input signal of each subsequent component of the system depends on the previous one.

Modules $NM_1 - NM_4$ were implemented in the form of a multilayer feedforward artificial neural network. Backpropagation was used as a learning algorithm.

It was experimentally determined that the presence of two hidden layers in each specified module is optimal. However, the problem of determining the exact number of neurons of the hidden layers of the artificial neural network does not have an unambiguous solution. The optimal number of neurons depends on many factors. To simplify the process of choosing the architecture of the neural network, a variety of metrics can be used that will allow to evaluate the problem that has to be solved and obtain an approximate number of neurons.

According to B. Widrow [25], the required number of neurons can be determined using the inequality for estimating weight coefficients in an artificial neural network, which is based on the size of training sample:

$$\frac{N_y N_p}{1+\log_2(N_p)} \le N_w < N_y\left(\frac{N_p}{N_x}+1\right)(N_x+N_y+1)+N_y, \quad (1)$$

where N_y is the dimension of the output signal of the network; N_p is the number of training sample instances; N_w is the number of weights in the network; N_x is the dimension of the input signal of the network.

It follows that the minimum number of neurons in the artificial neural network is [26]

$$N_{\mathrm{H}} = \frac{N_w}{N_x+N_y}. \quad (2)$$

By substituting the limiting values of N_w calculated by formula (1) into formula (2), the minimum ($N_{\mathrm{H}}^{\min}$) and maximum ($N_{\mathrm{H}}^{\max}$) number of neurons in the hidden layer of the network were determined [26].

In addition to the size of training sample, in order to determine the size of the network, the complexity of the approximation of the function is estimated. For this purpose, a sample-based estimation of the Lipschitz constant can be used [26]:

$$L_{\{x,\ y\}} = \max_{i\neq j}\sqrt{\frac{\sum_{k=1}^{K}\left(y_k^i - y_k^j\right)^2}{\sum_{k=1}^{K}\left(x_k^i - x_k^j\right)^2}}, \quad (3)$$

where y are the values of the required output signals of the neural network; x are the values of the input signals of the network.

According to formula (3), the Lipschitz constant for the original data set was calculated – $L_{\{x,\ y\}}$=36.59.

The estimate of the Lipschitz constant of a multilayer artificial network with a sigmoidal activation function is calculated by the following formula [26]:

$$L_S \leq c^k \sqrt{N_x N_y} \prod_{i=1}^{k-1} N_{\mathrm{H}_i}, \tag{4}$$

where c is the activation function parameter; k is the number of layers; N_x is the number of incoming signals; N_y is the number of output signals; N_{H_i} is the number of neurons on the i-th layer.

A neural network is capable of solving the problem of approximating a given tabular function under the condition $L_S \geq L_{\{x,\ y\}}$. Then the number of hidden neurons should not be less than [26]

$$N_{\mathrm{H}^L_{min}} = \frac{L_{\{x,\ y\}}}{c^k \sqrt{N_x N_y}}. \tag{5}$$

Using formulas 2 and 5, the minimum required number of hidden neurons of the artificial neural network for each module of the system was determined. Original values and the obtained results of calculations using the Widrow inequality and the sample-based estimation of the Lipschitz constant are presented in Table 3.

Table 3

Results of estimating the number of neurons

Module	Using Widrow inequality							Using sample-based estimation of the Lipschitz constant			
	N_x	N_y	N_p	$N_w^{\min}$	$N_w^{\max}$	$N_{\mathrm{H}}^{\min}$	$N_{\mathrm{H}}^{\max}$	$L_{\{x,\ y\}}$	c	k	$N_{\mathrm{H}^L_{\min}}$
NM_1	16	1	122	15	156	2	9	36.59	1	2	9
NM_2	17	1	122	15	156	2	9	36.59	1	2	9
NM_3	18	1	122	15	157	2	8	36.59	1	2	9
NM_4	19	1	122	15	157	2	8	36.59	1	2	8

Thus, the minimum number of neurons in the hidden layer of an artificial neural network will be [26]

$$N_{\mathrm{H}} = \max\left\{N_{\mathrm{H}}^{min}, N_{\mathrm{H}^L_{min}}\right\}. \tag{6}$$

Using the previously obtained estimates of the number of artificial neurons, the minimum number of hidden layer neurons for each neural network module was calculated using formula 6: for NM_1, NM_2 and NM_3 the number of neurons is 9, and for NM_4 – 8.

The hyperbolic tangent was used as the activation function for the neurons of the hidden layers in the modules NM_1 – NM_4 of the system. The activation function was not used in the neurons of the output layers.

Taking into account the presence of minor deviations in the data, methods with the advantages of robustness were used during training. In particular, AdaMax was used as the optimization algorithm for training the system's neural networks, which, due to the use of infinity norm, is more stable compared to the original Adam algorithm [27].

The Huber loss function was used to calculate the error of the neural network modules:

$$\rho(y^*, y) = \begin{cases} \frac{1}{2}(y^* - y)^2, & \text{для } |y^* - y| \leq k, \\ k|y^* - y| - \frac{1}{2}k^2, & \text{для } |y^* - y| > k, \end{cases} \tag{7}$$

where y^* is the required network response; y is the resulting network response; k is the function shape control parameter.

k affects the shape of the loss function: when the error value is large, the function assumes the behavior of the mean absolute error, when the values are small – of the mean squared error. Thus, k determines the sensitivity to outliers, minimizing too large values and smoothly optimizing small ones. However, this parameter requires fine tuning

According to J. Casella and R. L. Berger [28], sufficiently small values of the parameter k in the Huber loss function give a “median” estimate. Considering a sample containing outliers, it was concluded that with an increase in the value of the parameter k, the Huber estimate changes in the range between the median and the mean value. The authors interpret an increase in k as a decrease in robustness to outliers.

Based on the analysis, the median of absolute deviations during training of neural networks was used to determine the tuning parameter k.

Based on the obtained estimates of the minimum number of artificial neurons in the hidden layers of each model (Table 3), their initial number in each of the four modules was 9. During the training of each individual network, the number of neurons was refined by applying a constructive algorithm. Table 4 shows the obtained values of their optimal number in each module.

Table 4

The optimal number of neurons determined during training

Module	1st hidden layer	2nd hidden layer
NM_1	9	9
NM_2	9	9
NM_3	11	11
NM_4	10	10

To evaluate the performance of the overall system, several estimates were used: the mean squared error (MSE) and the mean absolute error (MAE). The MSE value was 0.1, and the MAE was 0.2. The obtained values of both errors are low which indicates a fairly good generalization ability of the system.

Table 5 shows an example of the result of strategy formulation for the 2024 Miami Grand Prix and a comparison with the real strategies chosen by the winning teams.

Table 5

Comparison of real and obtained strategies for the 2024 Miami Grand Prix

Strategy (real/obtained)	The place taken by the driver	Start	Pit 1	Pit 2	Pit 3
Real	1	2	1	0	0
Real	2	2	1	0	0
Real	3	2	1	0	0
Obtained	–	1.884	1.182	0.213	0.283

Based on the research, an application was implemented that allows users, team strategists, to perform the tasks set for them: view information, statistics, and formulate race strategies. The user interface is convenient and informative: it provides tips through text and pop-up tools, uses color coding, and has high contrast to increase readability. Figures 3 and 4 show the interface of the various application windows.

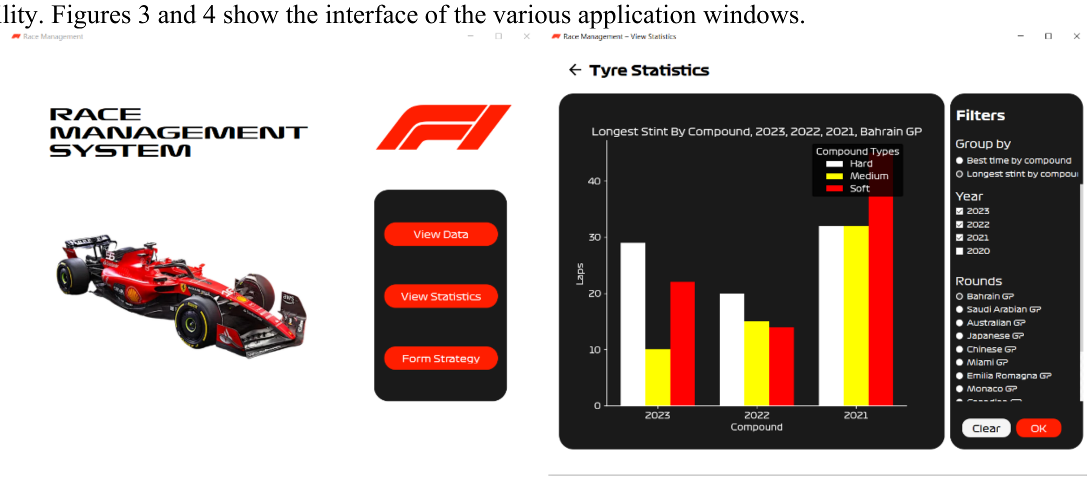

Fig. 3. Main window and statistics viewing window interfaces

Figure 4 shows the interface of the strategy window (the track was selected in advance to demonstrate the functionality) and the output window displaying the result of its formulation.

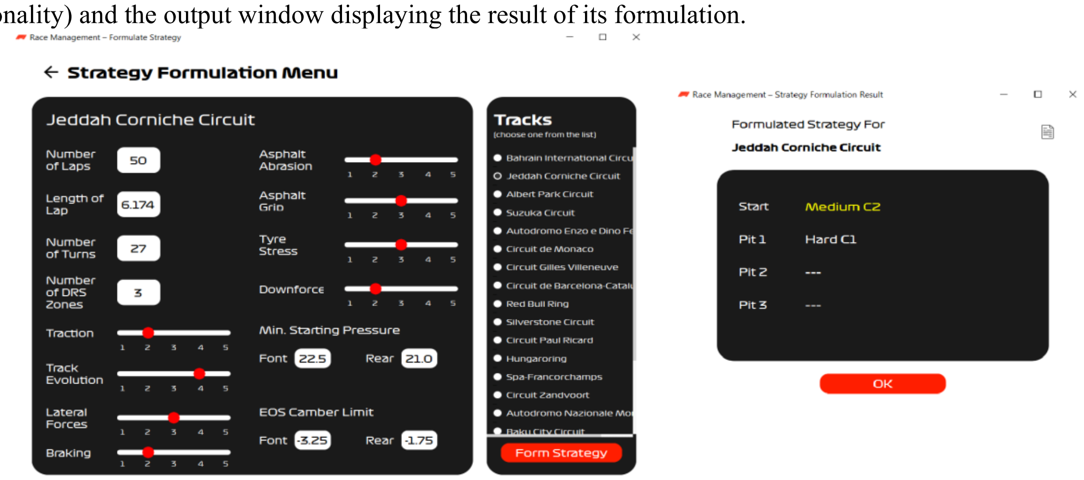

Fig. 4. Interfaces of strategy formation and output windows

Conclusions

Thus, as a result of the study, a model was built consisting of four modules – feedforward artificial neural networks. Based on the analysis conducted according to the results of the principal component analysis, a robust optimization method and a loss function were used when training artificial neural networks. This helped to offset the influence of minor deviations contained in the original data. The considered functions for estimating the optimal number of neurons in the hidden layers of neural networks made it possible to simplify and accelerate the process of determining their architectures.

The system of neural network modules obtained as a result of training shows a sufficiently small error in the tests (0.1) and allows the formulation of a strategy for the Formula One car racing series close in value to the real one determined by the team strategists.

The use of such a system will make it possible to determine a basic strategy plan, the use of which will subsequently reduce the time for making decisions, which in turn will contribute to achieving higher results in the race.

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