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METHODS OF ELECTROCARDIOGRAM CLASSIFICATION AND THEIR MATHEMATICAL MODEL IN THE FORM OF A CYCLIC DISCRETE RANDOM PROCESS

This paper presents an advanced approach to modeling electrocardiogram signals by integrating amplitude-time characteristics to obtain novel and informative features for cardiac diagnostics. Based on a systematic analysis of 426 scientific publications from the Scopus database (2014-2024), we identified a significant transformation in methodological approaches from classical signal processing to the implementation of modern artificial intelligence technologies. A geographical analysis of publications revealed that India, the United States, and Germany led the research in this field, with 78, 64, and 37 publications, respectively. The thematic distribution of works encompasses computer science (23.3%), engineering (22.4%), medicine (13.8%), and related fields, highlighting the interdisciplinary nature of these studies. We identified key developmental directions in electrocardiogram signal processing methods, including the improvement of filtering algorithms and data preprocessing, the development of new methods for extracting informative features, and the creation of hybrid classification systems. Particular attention was paid to integrating machine learning methods with traditional approaches to electrocardiogram signal analysis. The research demonstrated that while convolutional neural networks exhibit high classification accuracy (>95%) for cardiac arrhythmias, there remains a need for mathematical models that account for both rhythmic and morphological features of ECS signals. We propose a model of cyclic discrete random process with a time rhythm function that incorporates amplitude values of characteristic ECS peaks (P, Q, R, S, T). This model effectively captures the inherent cyclicity of ECS signals while accounting for their stochastic variations and corresponding amplitude values of diagnostic waves. The model distinguishes between regular and irregular cardiac rhythms. Experimental validation using ECS signals from healthy individuals and patients with extrasystole demonstrates the model's sensitivity to changes in cardiovascular system states. The time rhythm function, considering amplitude, exhibits distinctive patterns that effectively differentiate between normal and pathological conditions. The proposed mathematical framework expands the analytical toolset for ECS signal processing and provides a foundation for developing new diagnostic algorithms with enhanced accuracy for cardiac rhythm disorders.

Keywords: electrocardiogram modeling, cyclic random process, amplitude-time characteristics, cardiac signal analysis, mathematical modeling, time rhythm function, cardiac diagnostics, ECS peak morphology, stochastic signal processing, cardiovascular disease detection, pattern recognition, cardiac arrhythmia, extrasystole, cardiac rhythm disorders, biomedical signal analysis, signal classification, artificial intelligence (AI), machine learning system (MLS), neural network.

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МЕТОДИ КЛАСИФІКАЦІЇ ЕЛЕКТРОКАРДІОСИГНАЛІВ ТА ЇХ МАТЕМАТИЧНА МОДЕЛЬ У ВИГЛЯДІ ЦИКЛІЧНОГО ДИСКРЕТНОГО ВИПАДКОВОГО ПРОЦЕСУ

У статті представлено вдосконалений підхід до моделювання електрокардіосигналів шляхом інтеграції амплітудно-часових характеристик для отримання нових інформативних ознак у кардіодіагностиці. На основі систематичного аналізу 426 наукових публікацій з бази даних Scopus (2014-2024 рр.) виявлено значну трансформацію методологічних підходів від класичного опрацювання сигналів до впровадження сучасних технологій штучного інтелекту. Географічний аналіз публікацій виявив лідерство Індії (78 публікацій), США (64 публікації) та Німеччини (37 публікацій) у дослідженнях цього напряму. Тематичний розподіл робіт охоплює комп'ютерні науки (23,3%), інженерію (22,4%), медицину (13,8%) та суміжні галузі, що підкреслює міждисциплінарний характер досліджень. Визначено основні напрями розвитку методів опрацювання електрокардіосигналів: вдосконалення алгоритмів фільтрації та попередньої обробки даних, розробка нових методів виділення інформативних ознак, створення гібридних систем класифікації. Особливу увагу приділено інтеграції методів машинного навчання з традиційними підходами до аналізу електрокардіосигналів. Дослідження показало, що хоча згорткові нейронні мережі демонструють високу точність класифікації (>95%) серцевих аритмій, залишається потреба в математичних моделях, які враховують як ритмічні, так і морфологічні особливості ЕКС. Запропонована модель циклічного дискретного випадкового процесу з функцією часового ритму, яка включає амплітудні значення характерних піків ЕКС (P, Q, R, S, T). Ця модель ефективно відображає притаманну сигналам ЕКС циклічність, враховуючи при цьому їх стохастичні варіації та відповідні амплітудні значення діагностичних піків. Модель розрізняє регулярні та нерегулярні серцеві ритми Експериментальна перевірка на ЕКС здорових людей та пацієнтів з екстрасистолією демонструє чутливість моделі до змін стану серцево-судинної системи. Функція часового ритму з урахуванням амплітуди демонструє чіткі патерни, які ефективно диференціюють нормальні та патологічні стани. Запропонований математичний апарат розширює аналітичний інструментарій для обробки ЕКС і забезпечує основу для розробки нових алгоритмів діагностики порушень серцевого ритму з підвищеною точністю.

Ключові слова: моделювання електрокардіосигналів, модель, аналіз, класифікація, діагностика, алгоритм, циклічний випадковий процес, амплітудно-часові характеристики, аналіз кардіосигналів, математичне моделювання, часова функція ритму, кардіодіагностика, морфологія піків ЕКС, стохастична обробка сигналів, виявлення серцево-судинних *захворювань, розпізнавання образів, серцева аритмія, екстрасистолія, порушення серцевого ритму, біомедичний аналіз сигналів, класифікація сигналів, штучний інтелект (AI), система машинного навчання (MLS), нейронна мережа.*

Introduction

In modern conditions of diagnosing cardiovascular diseases of mankind, the timeliness and efficiency of the processing and analysis of electrocardio signals (ECS) is especially relevant. After all, cardiovascular diseases have a high prevalence and a high mortality rate in the world [1]. The latest tools of machine learning and artificial intelligence [2-6] open up new opportunities for improving methods for modeling, processing, and classifying electrocardio signals [7, 8]. Therefore, a systematic analysis of modern scientific research in this area is of particular importance to identify the main trends, promising directions, and methodological approaches to the analysis of CEN [9, 10]. The purpose of this study is to conduct an analytical review of scientific publications on methods of modeling, processing, and classification of electrocardio signals over the past ten years based on the data of the scientometric database Scopus.

Related works

To conduct an analytical review of scientific publications indexed in the Scopus scientometric database, regarding the analysis of methods for modeling, processing, and classifying ECS signals over the past ten years, a request for an extended search was formed:
TITLE-ABS-KEY("cardiac signal") AND (TITLE-ABS-KEY(model* method) OR TITLE-ABS-KEY(*cardiogram) OR TITLE-ABS-KEY(forecasting) OR TITLE-ABS-KEY(automated process*) OR TITLE-ABS-KEY(analysis) OR TITLE-ABS-KEY(classification) OR TITLE-ABS-KEY(diagnostic) OR TITLE-ABS-KEY(construction) OR TITLE-ABS-KEY(evaluation) OR TITLE-ABS-KEY(review) OR TITLE-ABS-KEY("computer system") OR TITLE-ABS-KEY("decision making") OR TITLE-ABS-KEY("expert system")) AND PUBYEAR > 2013 AND PUBYEAR < 2025 AND (LIMIT-TO (EXACTKEYWORD,"Signal Processing") OR LIMIT-TO (EXACTKEYWORD,"Algorithm") OR LIMIT-TO (EXACTKEYWORD,"Machine Learning") OR LIMIT-TO (EXACTKEYWORD,"Signal Encoding") OR LIMIT-TO (EXACTKEYWORD,"Algorithms") OR LIMIT-TO (EXACTKEYWORD,"Electrocardiography") OR LIMIT-TO (EXACTKEYWORD,"Convolutional Neural Network") OR LIMIT-TO (EXACTKEYWORD,"Biomedical Signal Processing") OR LIMIT-TO (EXACTKEYWORD,"Convolutional Neural Networks") OR LIMIT-TO (EXACTKEYWORD,"Signal Detection") OR LIMIT-TO (EXACTKEYWORD,"Classification (of Information)") OR LIMIT-TO (EXACTKEYWORD,"Neural Networks") OR LIMIT-TO (EXACTKEYWORD,"Electrocardiogram") OR LIMIT-TO (EXACTKEYWORD,"Deep Neural Networks") OR LIMIT-TO (EXACTKEYWORD,"Heart") OR LIMIT-TO (EXACTKEYWORD,"Numerical Methods") OR LIMIT-TO (EXACTKEYWORD,"Artificial Neural Network") OR LIMIT-TO (EXACTKEYWORD,"Artificial Intelligence")) AND (LIMIT-TO (SUBJAREA,"COMP") OR LIMIT-TO (SUBJAREA,"MEDI") OR LIMIT-TO (SUBJAREA,"ENGI") OR LIMIT-TO (SUBJAREA,"MATH") OR LIMIT-TO (SUBJAREA,"NEUR") OR LIMIT-TO (SUBJAREA,"DECI") OR LIMIT-TO (SUBJAREA,"HEAL") OR LIMIT-TO (SUBJAREA,"MULT"))

The results of an extended search in the Scopus scientometric database reflect a significant increase in the number of publications over the past ten years (Fig. 1). The total number of publications over the past decade amounted to 426 papers. The largest number of publications was in 2019 (47 papers), 2022 (49 papers), and 2024 (59 papers). ECS modeling, processing, and classification are done both by classical methods and with the help of artificial intelligence.

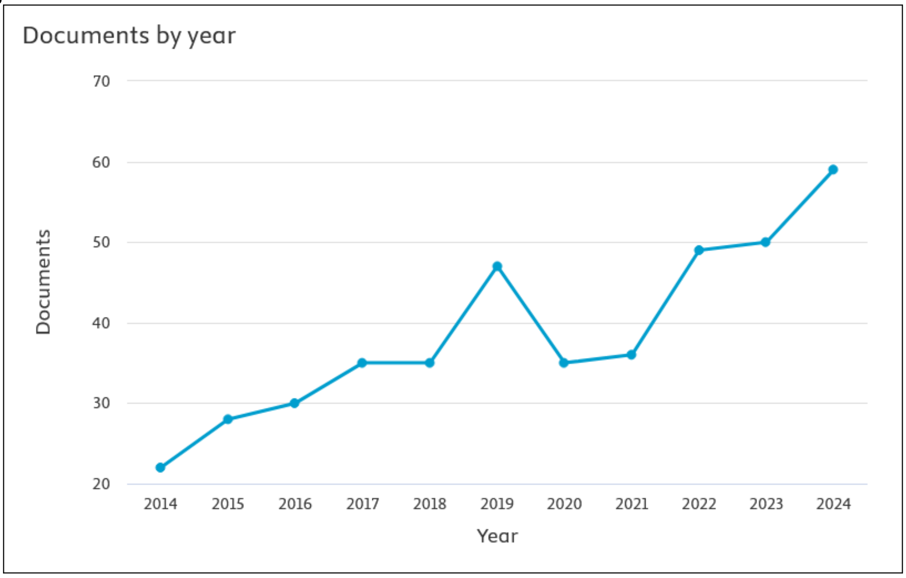

Fig. 1. Dynamics of publications in the Scopus scientometric database by years

After conducting a comprehensive review and analysis of the most relevant articles on methods for modeling, processing, and classifying cardiac signals, in particular electrocardiograms (ECS), the following can be noted. In the article by Feng C., the author looked at deep learning algorithms for the classification of ECSs, which he classified into three main types: convolutional neural networks, recurrent neural networks, and multimodal deep learning [7].

Ardeti V. A., Kolluru V. R., Varghese G. T., & Patjoshi R. K. reviewed state-of-the-art ECS signal processing techniques, ranging from traditional to AI-based approaches: using portable and wearable devices, the Internet of Things (IoT), and wireless technologies for remote patient monitoring [8].

The scientific paper by Fikri, M. R., Soesanti, I., & Nugroho, H. A. provides a comprehensive overview of ECS signal classification methods, discussing the various steps involved, including preprocessing, feature extraction, and classification methods such as MLP, K-NN, SVM, CNN, and RNN, and concluding that neural network techniques such as CNN and RNN are the best and most popular for ECS classification due to their high accuracy and low complexity [9].

Wasimuddin M., Elleithy K., Abuzneid A.-S., Faezipour M., & Abuzaghleh O. presented a comprehensive review of the literature on real-time ECS signal acquisition, pre-recorded clinical ECS data, ECS signal processing and denoise, detection of fiducial ECS points based on feature engineering and ECS signal classification, and comparative discussions among the studies reviewed [10]. The classification of ECS heart contractions into normal and abnormal was made by the authors using methods based on threshold values [19-20]. A modified adaptive threshold approach based on Pan-Tompkins [21] was introduced in [22]. DWT is also used to classify ECSs using Basic Component Analysis (PCA) and Independent Component Analysis (ICA) as described in [23]. However, the Multi-Model Decision Learning (MDL) algorithm achieved a better sensitivity of 100% in classifying the ECS as normal and abnormal when evaluated on the MIT-BIH arrhythmia dataset. However, the Multi-Model Decision Learning (MDL) algorithm achieved a better sensitivity of 100% in classifying the ECS as normal and abnormal when evaluated on the MIT-BIH arrhythmia dataset. These methods were summarized by Wasimuddin M., Elleithy K., Abuzneid A.-S., Faezipour M., & Abuzaghleh O. in Table 1.

Table 1

Classification of ECS according to traditional algorithms

Class [Ref.]	Algorithm	Performance Metrics	Dataset
Normal, Abnormal [24]	Modified Tompkins	$99{,}51\%_{acc}$, 0,0049err	MITDB [27]
Normal, Abnormal [22]	Adaptive-profiling	$97{,}47\%_{acc}$, $99{,}8\%_{sen}$, $99{,}79\%_{ppv}$, 0,0258err	
Normal, Abnormal [23]	PCA, ICA	$99{,}28\%_{acc}$, $97{,}97\%_{sen}$, 99,21ppv	
Normal, Abnormal [25]	MDL	$93{,}33\%_{acc}$, $100\%_{sen}$, $81{,}81\%_{spe}$, $90{,}47\%_{ppv}$	
Normal, Abnormal [19]	Threshold-based	$97{,}6\%_{acc}$, $97{,}3\%_{sen}$, $98{,}8\%_{spe}$	PTBDB [28]
Normal, Ischemic [20]	Threshold-based	$98{,}12\%_{sen}$, $98{,}16\%_{spe}$	ESCDB [29]
Normal, Abnormal [26]	Regression	$97\%_{sen}$, $88\%_{spe}$, $97\%_{ppv}$	Sample Collection

The article by authors Martinek R., Ladrova M., Sidikova M., Jaros R., Behbehani K., Kahankova R., & Kawala-Sterniuk A. provides a comprehensive overview of advanced signal processing techniques for cardiac bioelectrical signals, spanning both classical and advanced approaches. Key methodological components include digital filtering (both adaptive and non-adaptive), signal decomposition techniques such as wavelet transform and blind source separation, and an overview of various signal processing techniques applicable for clinical use [11].

The article by Abdulla L. A., & Al-Ani M. S. provides a comprehensive overview of the most common methods used to classify ECS signals, including ANN, CNN, DWT, SVM, and KNN, and discusses their performance, advantages, and limitations. The methodology is to pre-process the ECS signal for noise removal using methods such as low and high pass filters, extracting features from the ECS signal using discrete wavelet transform (DWT), principal component analysis (PCA) and independent components (ICA), classifying ECS signals using a variety of machine learning techniques, including artificial neural networks (ANNs), convolutional neural networks (CNNs), reference vector machines (SVM), decision trees (DT), k-nearest neighbors (KNN), and linear discriminant analysis (LDA) [12].

The authors Lupenko S. A., Sverstiuk A. S., Stadnyk N. B., Zozulia A. M., and Orobchuk O. investigate methods for modeling different types of cardiac signals (electrical, magnetic, and acoustic/mechanical) using the theory of cyclic random functions. In particular, they apply a cyclic random process and a vector of cyclic rhythmically related random processes. The study includes the development of a structure of statistical assessments of the probabilistic characteristics of cardiac signals, spectral analysis of these signals, and the determination of informative features for computer systems for functional diagnostics of heart conditions [13-14].

A study by Kapatsila, R., & Sverstiuk, A. is dedicated to the development of a decision tree-based system for predicting cardiovascular disease. The model uses 12 key medical indicators, including demographics, ECS results, and other physiological parameters. As a result, a correlation matrix of risk factors was created, a decision tree was developed and tested, the quality of classification was assessed through an error matrix, and ROC analysis showed high efficiency of the model (AUC = 0.9) [15].

The article by Mishra A., Bhusnur S., Mishra S. K., & Singh P. discusses a method for modeling ECS using parametric quart splines to generate synthetic data. The study includes the classification of normal and abnormal sinus rhythms using three machine-learning techniques in the Orange environment. The assessment of the quality of synthetic signals showed a high correlation coefficient (0.974) between synthetic and real signals [16].

Edla S., Kovvali N., & Papandreou-Suppappola A. developed a Bayesian classifier that achieved an accuracy of 90% when using the adaptive method and 98% when applying IMM and SMCMC to classify different types of arrhythmias [17].

The article Lytvynenko I., Horkunenko A., Kuchvara O., Palaniza Y. presents an automated method of ECS processing based on signal simulation as a cyclic random process. The developed system includes methods for segmenting the ECS signal, estimation of rhythmic structure, and statistical data processing. The results obtained indicate the reliability of the developed model and the possibility of its use in diagnostic systems to predict the risk of cardiovascular diseases. Based on the proposed mathematical models and methods, the authors substantiate which characteristics of signals are the most informative for diagnosing the state of the heart [18].

Fig. 2 shows a word cloud that displays all the keywords of the authors of the Scopus database publications on the topic under study.

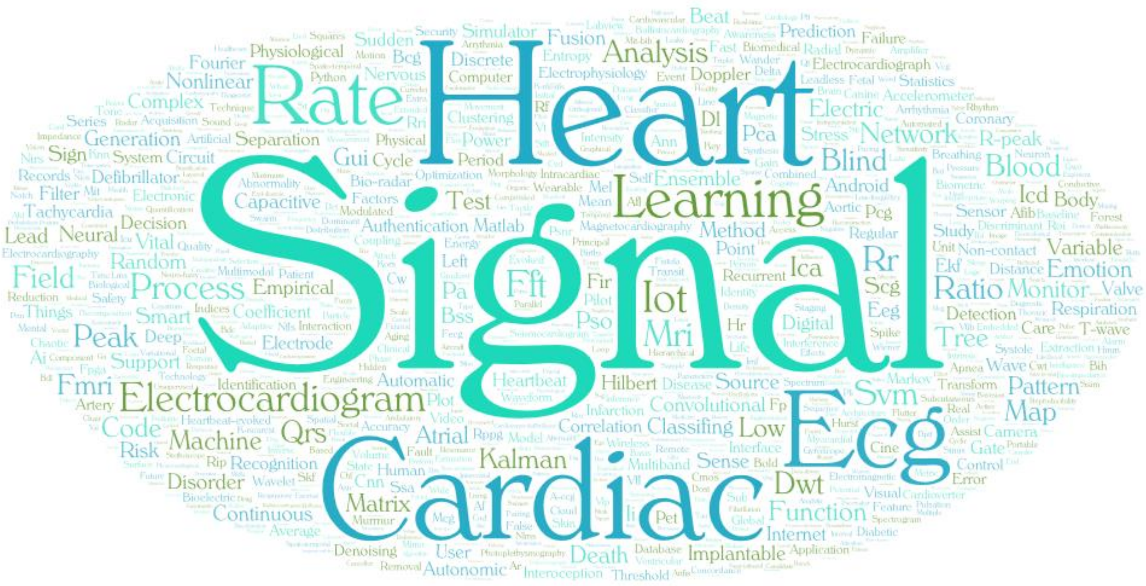

Fig. 2. Keyword cloud used in authors' posts

The number of papers per year by publication sources has been growing rapidly since 2022 in IEEE Access, while Proceedings of The Annual International Conference of The IEEE Engineering in Medicine and Biology, Society EMBS, Sensors, Sensors Switzerland, Biomedical Signal Processing and Control show a decline or the same level (Fig. 3).

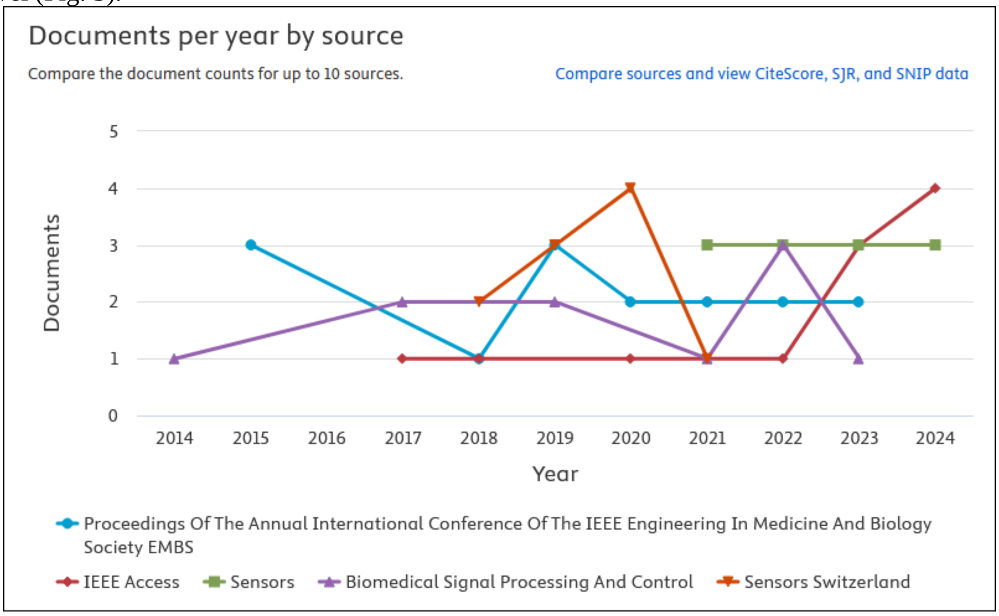

Fig. 3. Number of documents per year by sources of publications

As for the authors, such scientists as Mohanty M.N., Mohapatra S.K. – 7 publications each, Peris-Lopez P. – 6 publications, Camara C. – 5 publications, Blanco-Velasco M., Castells F., Fonseca P., Gouveia C., Millet J. – 4 publications each and Bagheri N. – 3 publications (Fig. 4).

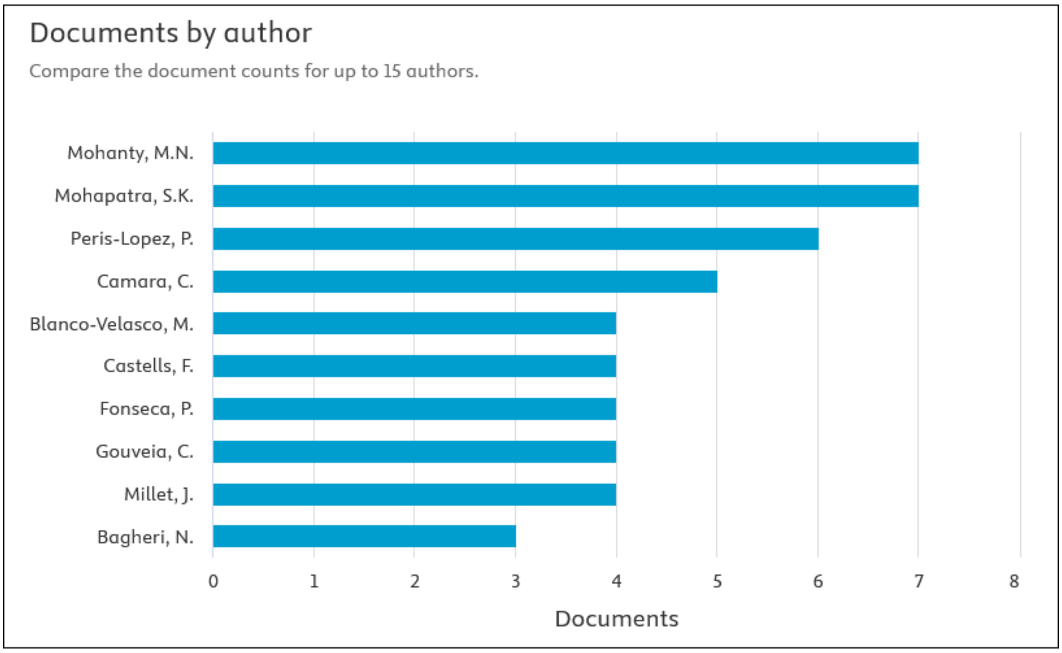

Fig. 4. Number of publications in the Scopus database by authors

The largest number of publications by country or territory belongs to India – 78, the USA – 64, Germany – 37, China – 30, the United Kingdom – 28, Italy – 26 and Spain – 25. Thus, mainly developed countries of the world are conducting scientific developments in the prediction and treatment of cardiovascular diseases (Fig. 5).

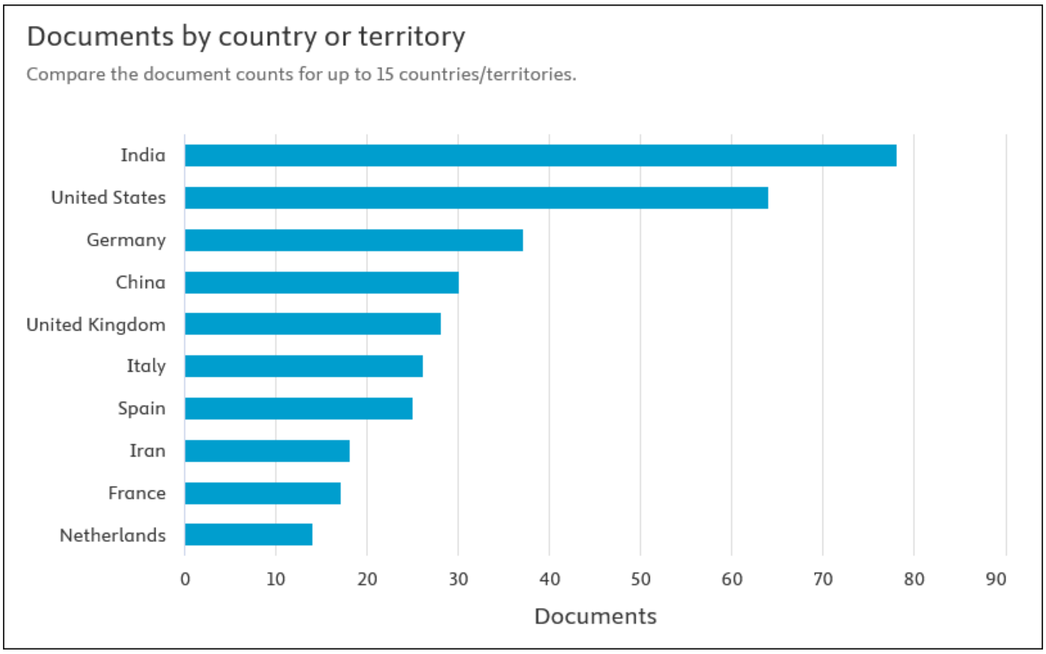

Fig. 5. Number of publications by country or territory

By types of documents, articles (57.5%), conference proceedings (36.2%) and review articles (4.0%) prevailed among scientific papers (Fig. 6).

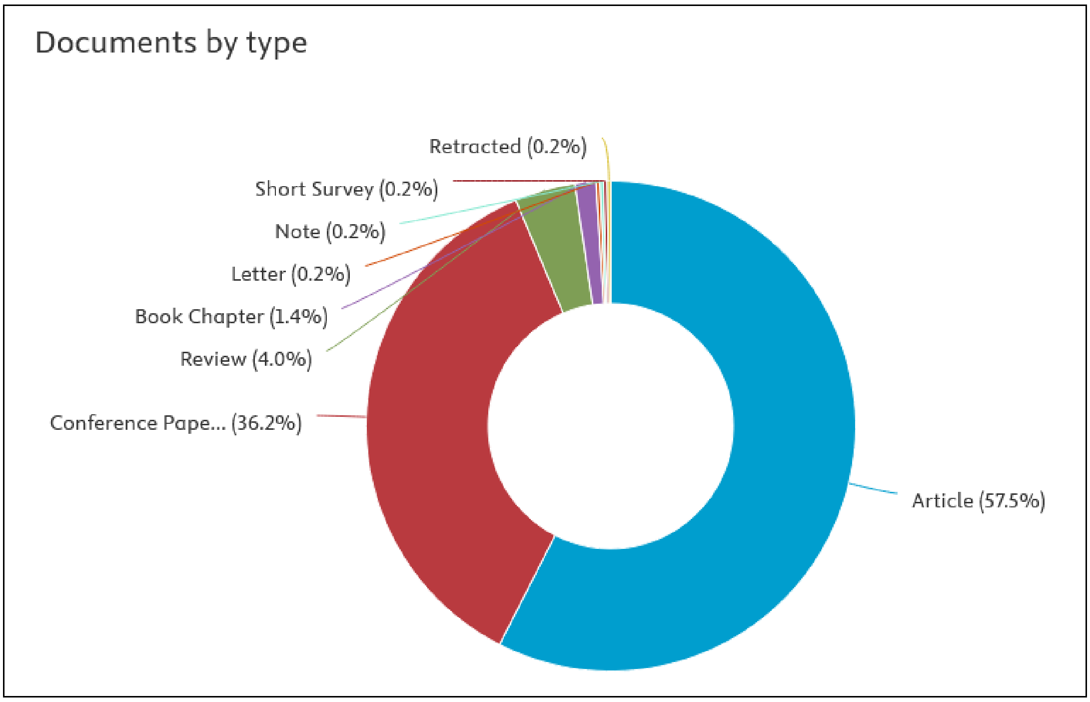

Fig. 6. Number of works by type of publications

According to the subject area, publications were distributed: computer science – 23.3%, engineering – 22.4%, medicine – 13.8%, physics and astronomy – 7.0%, mathematics – 6.0%, biochemistry, genetics and molecular biology – 5.9%, and neurology – 3.2% (Fig. 7).

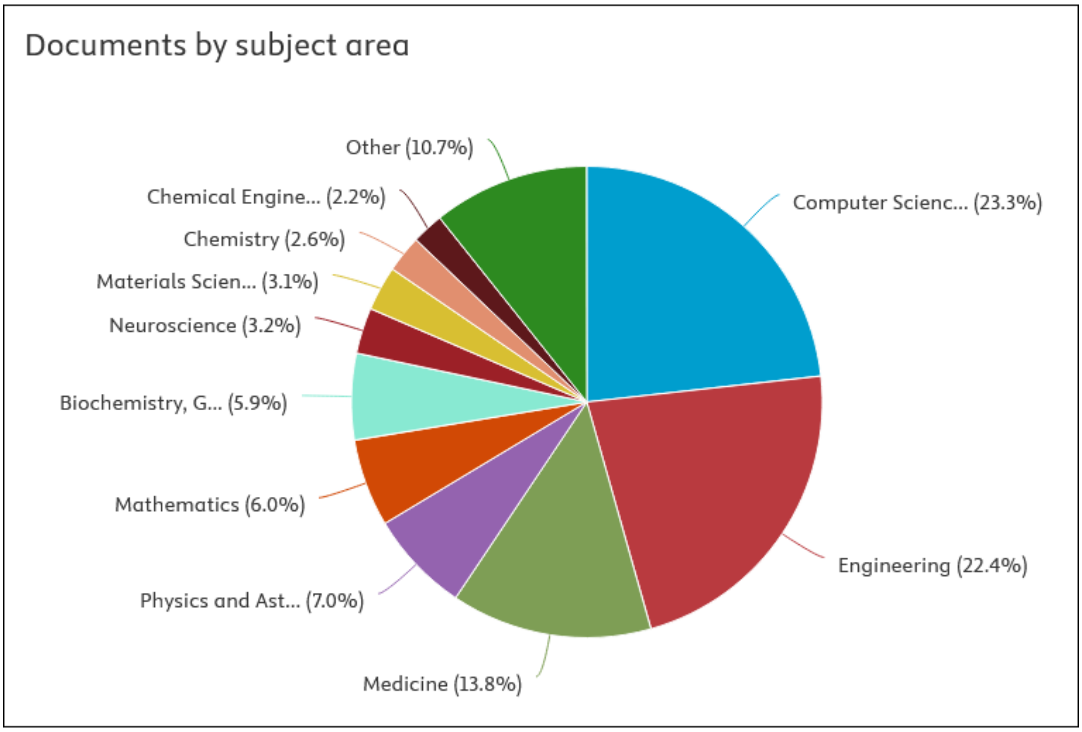

Fig. 7. Number of publications by subject area

In terms of the number of publications, Universidad Rey Juan Carlos, Siksha O Anusandhan Deemed to be University – 8 documents each, and Universidad Carlos III de Madrid – 7 documents are in the lead (Fig. 8).

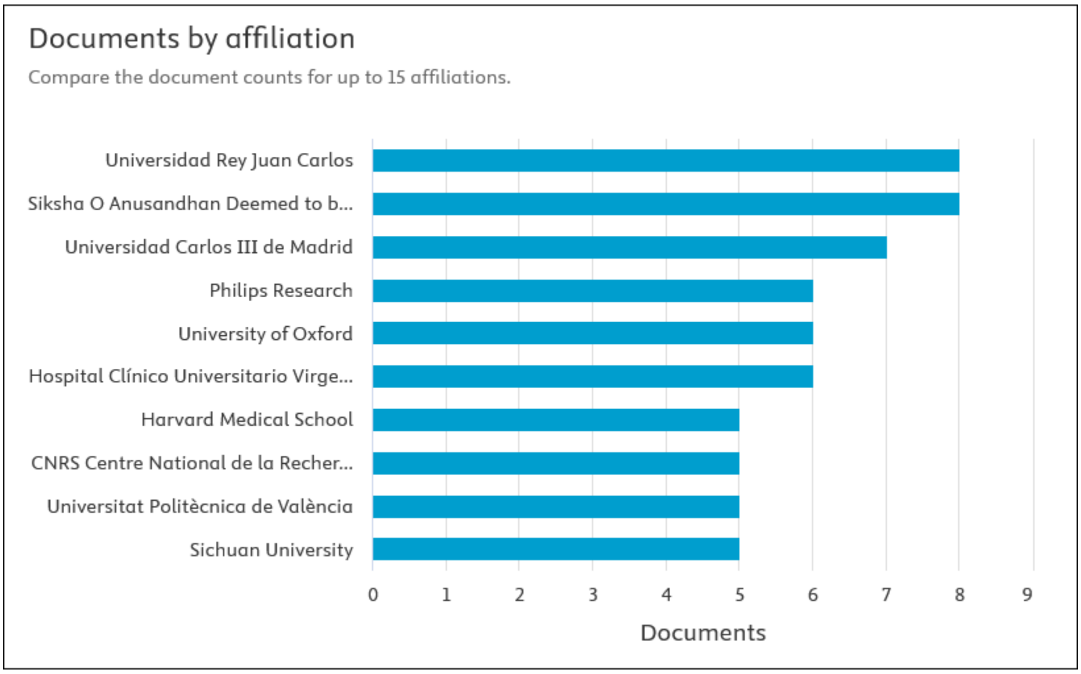

Fig. 8. Number of publications by affiliation to organizations

Model of cyclic discrete random process with time rhythm function incorporating Amplitude values of characteristic electrocardiogram peaks

Based on the systematic review of publications on electrocardiogram signal modeling and classification presented above, we propose a model of a cyclic discrete random process with a time rhythm function that incorporates the amplitude values of characteristic ECS peaks. This model accounts for both the time rhythmic functions of ECS and the amplitude values of the P, Q, R, S, and T peaks.

Building upon [30], we describe the fundamental properties of cyclic random processes, including the definitions of the rhythm function and cyclic random processes with continuous parameters. In general, the mathematical model of a cyclic signal represents a random process *ξ (ω, t), where ω* ∈ ***Ω*** *and t* ∈ ***R*** *(ξ:* ***R*** *→ L2(Ω, P)),* defined on a probability space ***(Ω, F, P)*** and the set ***R*** of real numbers. The argument *t* can be interpreted physically as a spatial or time coordinate, while the range represents a space of random variables defined on the same probability space (Ω, F, P).

The author [30] defines a discrete random process $\xi(\omega, t_{ml}), \omega \in \boldsymbol{\Omega}, t_{ml} \in \boldsymbol{D}$, which is called a cyclic discrete random process if there exists a discrete function $T(t_{ml}, n)$ that represents distances between in-phase samples l and satisfies the conditions of a rhythm function, such that finite-dimensional vectors $\left(\xi(\omega, t_{m_1 l_1}), \xi(\omega, t_{m_2 l_2}), \ldots, \xi(\omega, t_{m_k l_k})\right)$ and $\xi\left(\omega, t_{m_1 l_1} + T\left(t_{m_1 l_1}, n\right)\right)$, $\xi\left(\omega, t_{m_2 l_2} + T\left(t_{m_2 l_2}, n\right)\right)$, ..., $\xi\left(\omega, t_{m_k l_k} + T\left(t_{m_k l_k}, n\right)\right)$, $n \in \boldsymbol{Z}$, for all integers $k \geq 1$ are stochastically equivalent in the broad sense.

The domain $\boldsymbol{D} = \left\{t_{ml},\ m \in \boldsymbol{Z},\ l = \overline{1, L},\ L \geq 2\right\}$ is the domain of definition of the discrete cyclic random process $\xi(\omega, t_{ml})$, where *m* is the cycle number of the cyclic random process, and *l* is the sample number of the discrete random process within its *m*-th cycle [30].

Thus, for a discrete cyclic random process, the family of its distribution functions satisfies the following equalities:

$$F_{k_\xi}(x_1, \ldots, x_k, t_{m_1 l_1}, \ldots, t_{m_k l_k}) = F_{k_\xi}(x_1, \ldots, x_k, t_{m_1 l_1} + T(t_{m_1 l_1}, n), \ldots, t_{m_k l_k} + T(t_{m_k l_k}, n)),$$
$$x_1, \ldots, x_k \in \mathbf{R}, t_{m_1 l_1}, \ldots, t_{m_k l_k} \in \mathbf{D}, n \in \mathbf{Z}, k \in \mathbf{N}. \quad (1)$$

When conducting cardiac diagnostics, detailed analysis of time and amplitude changes in ECS parameters is crucial. Therefore, we propose a model of cyclic discrete random process with a time rhythm function that incorporates amplitude values of characteristic ECS peaks.

Our proposed model characterizes the ECS as a cyclic discrete random process $\xi(\omega, t_{mi}), \omega \in \boldsymbol{\Omega}, t_{mi} \in \boldsymbol{D}$. Here, t_{mi} refers to the extreme values of the characteristic peaks *P, Q, R, S, T* throughout the cardiac cycle, varying according to the respective lead in either normal or pathological states: *i = extremum {P* (t_{mi}, n)*; Q* (t_{mi}, n)*; R* (t_{mi}, n)*; S* (t_{mi}, n)*; T* (t_{mi}, n)*}*. This methodology enables a mathematical representation of the natural cyclicity of ECS signals while accounting for their stochastic variations along with the amplitude values of the analyzed signal peaks. A crucial component of our model is the rhythm function, which encapsulates the amplitude values of the characteristic ECS peaks $T(t_{mi}, n)$. This function qualifies as a rhythm function when finite-dimensional vectors

$(\xi(\omega, t_{m_1 i_1}), \xi(\omega, t_{m_2 i_2}), \dots, \xi(\omega, t_{m_k i_k}))$ i $(\xi\left(\omega, t_{m_1 i_1} + T(t_{m_1 i_1}, n)\right)$, $\xi\left(\omega, t_{m_2 i_2} + T(t_{m_2 i_2}, n)\right), \dots, \xi\left(\omega, t_{m_k i_k} + T(t_{m_k i_k}, n)\right)$, $n \in \boldsymbol{Z}$, for all integers $k \geq 1$ are stochastically equivalent in the broad sense.

Thus, for the discrete cyclic random process, the family of its distribution functions satisfies the following equalities:

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$$x_1, \dots, x_k \in \boldsymbol{R}, t_{m_1 i_1}, \dots, t_{m_k i_k} \in \boldsymbol{D}, n \in \boldsymbol{Z}, k \in \boldsymbol{N}. \quad (2)$$

The function $T(t_{m_k i_k}, n)$ serves as a time rhythm function that incorporates amplitude values of characteristic ECS peaks and defines the regular patterns of intervals between amplitude values of corresponding ECS peaks. The proposed model distinguishes between regular and irregular cardiac rhythms: when $T(t_{m_k i_k}, n) = n \cdot T(t_{m_k i_k})$, it represents a cyclic process with regular rhythm that accounts for amplitude values of characteristic ECS peaks, where as $T(t_{m_k i_k}, n) \neq n \cdot T(t_{m_k i_k})$ corresponds to irregular rhythmic patterns.

In practical application, we analyzed ECS under conditions of normal cardiac function and in patients with extrasystole using the cyclic discrete random process model with a time rhythm function that incorporates amplitude values of characteristic electrocardiogram peaks. Figure 9 provides a graphical representation of a healthy patient's ECS (conditional normal). The time function (diagnosis: conditional normal) of the rhythm taking into account the amplitude values of the characteristic peaks of ECS $T(t_{mP}, 1)$ is shown in Figure 10, $T(t_{mR}, 1)$ in Figure 11, and $T(t_{mT}, 1)$ in Figure 12.

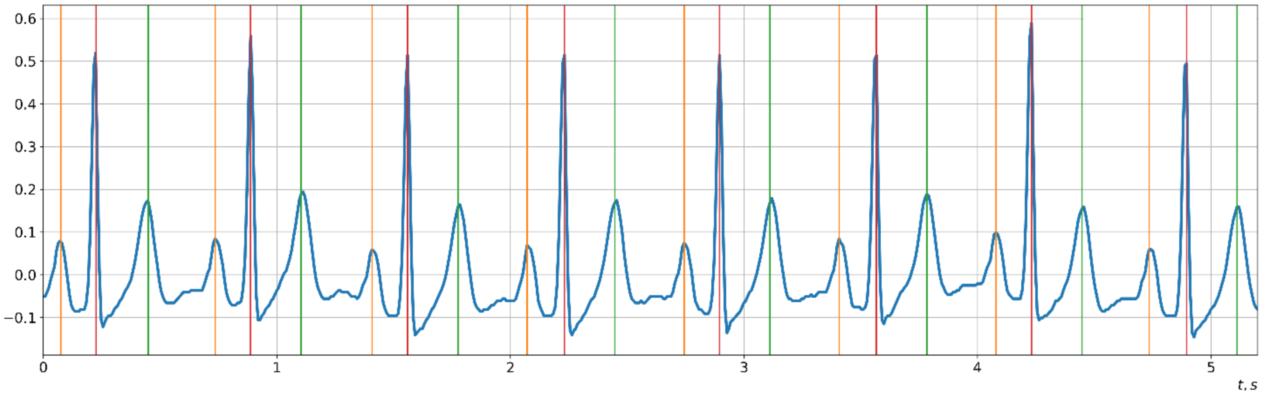

Fig. 9. Graphical representation of the ECS of a healthy patient (diagnosis: conditional normal)

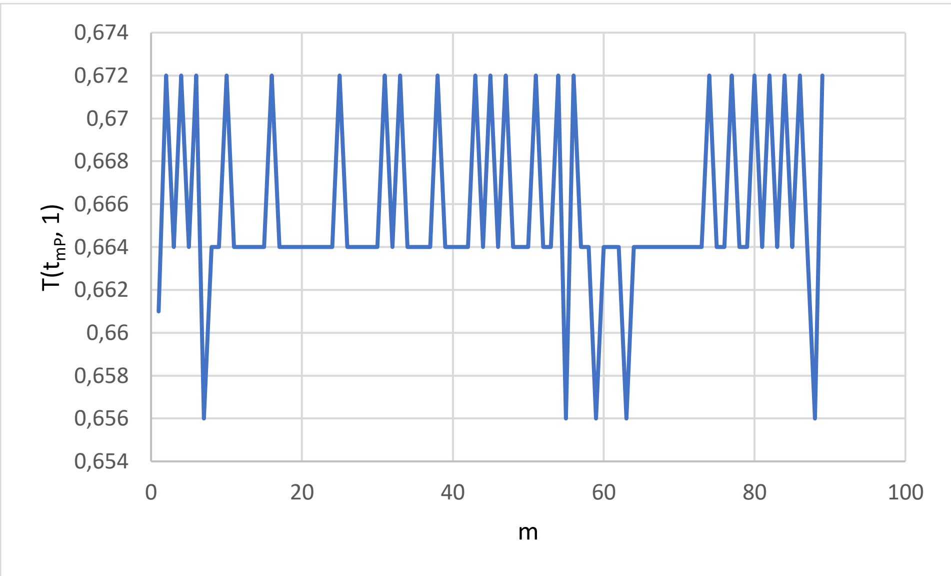

Fig. 10. The time function of the rhythm taking into account the amplitude values of the characteristic peaks of ECS T(tmP, 1) (diagnosis: conditional normal)

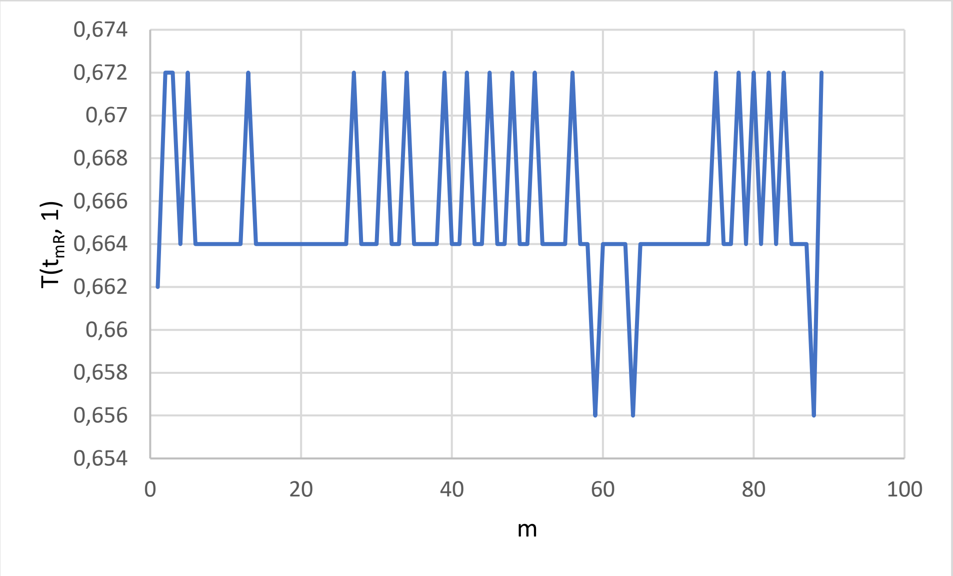

Fig. 11. The time function of the rhythm taking into account the amplitude values of the characteristic peaks of ECS T(tmR, 1) (diagnosis: conditional normal)

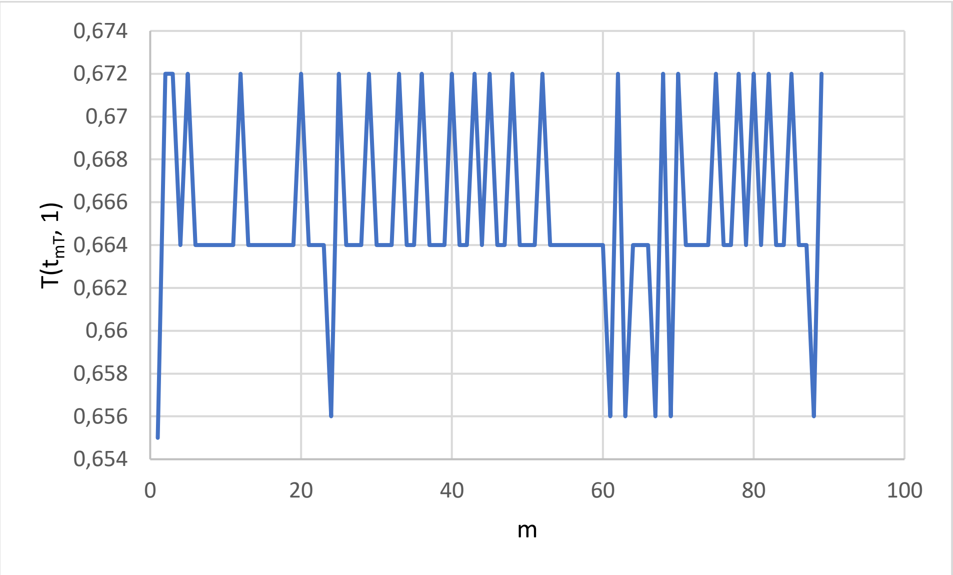

Fig. 12. The time function of the rhythm taking into account the amplitude values of the characteristic peaks of ECS T(tmT, 1) (diagnosis: conditional normal)

A graphical representation of the ECS from a patient with cardiac pathology (diagnosis: extrasystole) is shown in Figure 13. The time function (diagnosis: extrasystole) of the rhythm taking into account the amplitude values of the characteristic peaks of ECS $T(t_{mP}, 1)$ is shown in Figure 14, $T(t_{mR}, 1)$ in Figure 15, and $T(t_{mT}, 1)$ in Figure 16.

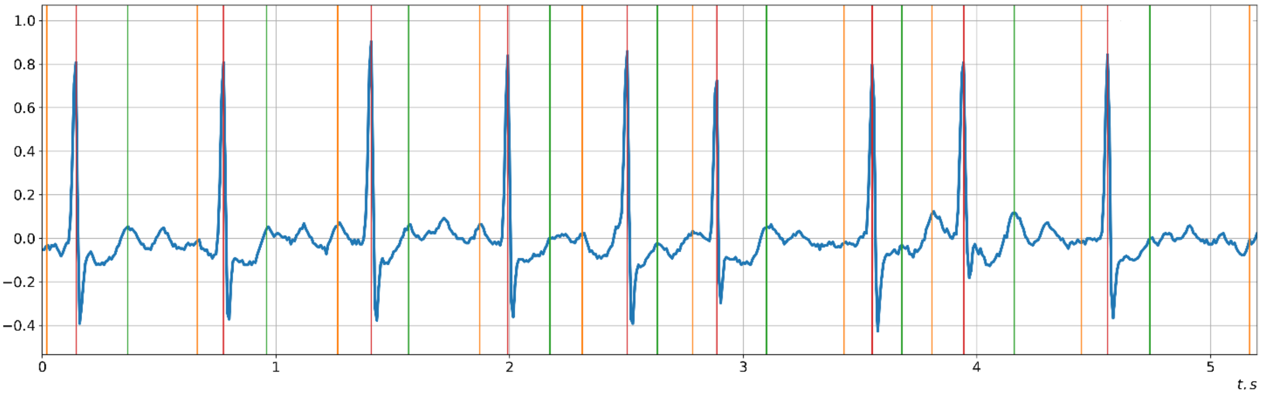

Fig. 13. Graphical representation of the ECS of a patient with cardiac pathology (diagnosis: extrasystole)

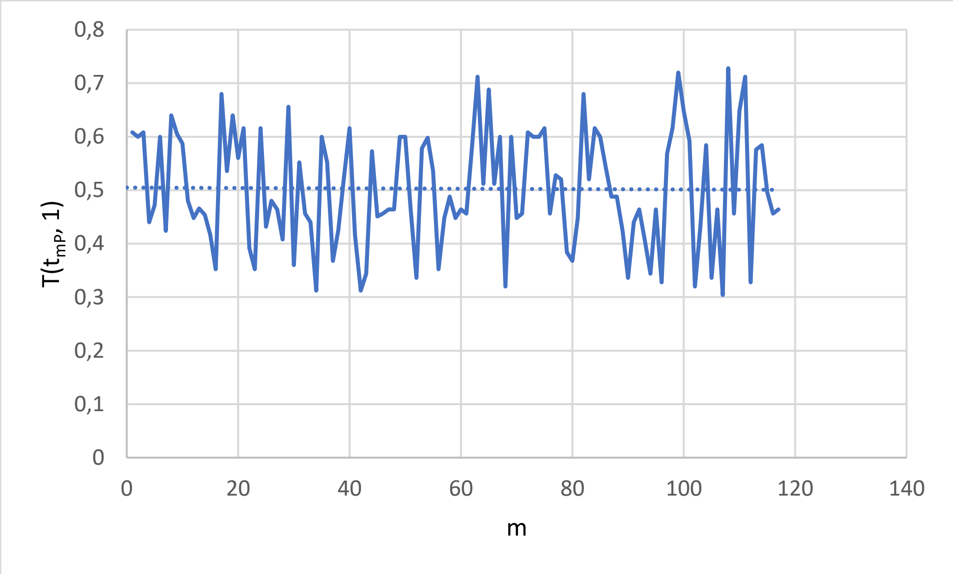

Fig. 14. The time function of the rhythm taking into account the amplitude values of the characteristic peaks of ECS T(tmP, 1) (diagnosis: extrasystole)

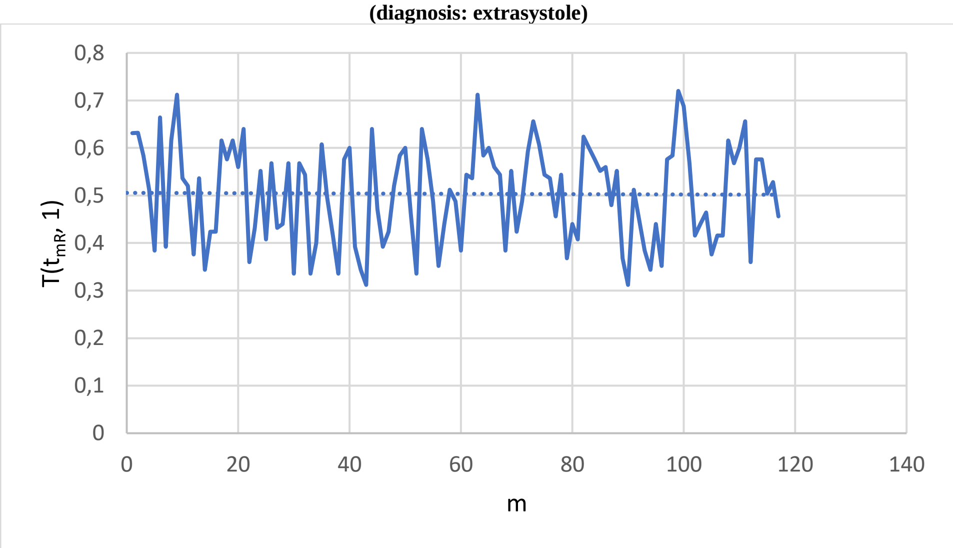

Fig. 15. The time function of the rhythm taking into account the amplitude values of the characteristic peaks of ECS T(tmR, 1) (diagnosis: extrasystole)

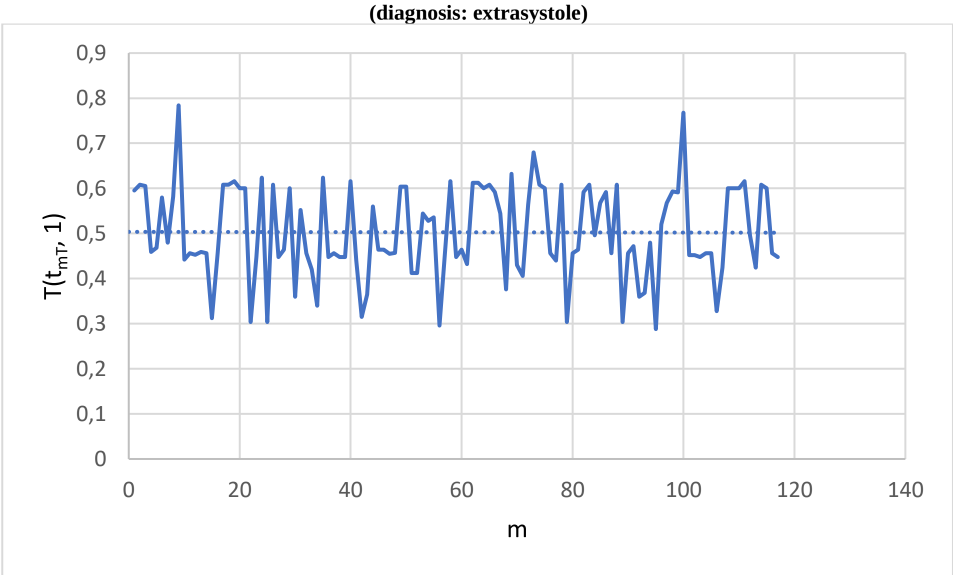

Fig. 16. The time function of the rhythm taking into account the amplitude values of the characteristic peaks of ECS T(tmT, 1) (diagnosis: extrasystole)

Based on the analysis of the time rhythm function incorporating amplitude values of characteristic electrocardiogram peaks, we can conclude that the proposed function demonstrates sensitivity to changes in the human cardiovascular system in both normal and pathological states. The results obtained provide a foundation for further scientific investigation of this rhythm function to identify informative diagnostic features of ECS within information technology systems for expert analysis of morphological and rhythmic characteristics of cardiac signals.

Conclusions

A distinctive feature of the proposed model is the logical development of the author's original concept [30], specifically adapted for electrocardiogram signal analysis. The key modification involves replacing the sampling index l with the index i, which corresponds to the extreme values of ECS peaks *(P, Q, R, S, T)*, thereby linking the mathematical model to specific physiological characteristics of the cardiac signal. The function $T(t_{ml}, n)$ now considers not only time but also the amplitude characteristics of ECS peaks, making the model more informative for cardiac diagnostics. All fundamental mathematical properties of the cyclic random process (stochastic equivalence, distribution functions) are preserved in the new model. Additionally, the model clearly distinguishes between regular and irregular cardiac rhythms, which may have direct clinical applications.

Thus, the proposed model expands the mathematical tools available for electrocardiogram signal analysis, especially in diagnosing cardiac rhythm disorders. It also establishes a foundation for the development of advanced diagnostic algorithms that improve accuracy in detecting and classifying cardiac arrhythmias. Future work will focus on extracting more informative diagnostic features from the proposed rhythm function and integrating this approach with machine learning methods to create hybrid systems that utilize the strengths of both mathematical modeling and artificial intelligence.

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