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INFORMATION TECHNOLOGY FOR ELECTROCARDIOGRAPHIC SIGNAL ANALYSIS BASED ON MATHEMATICAL MODELS OF TEMPORAL AND AMPLITUDE VARIABILITY

This paper presents an information technology for electrocardiographic signal analysis based on discrete mathematical models with temporal rhythm functions and amplitude variability of characteristic waves P, Q, R, S, T. A discrete mathematical model of the temporal rhythm function considering extreme amplitude values of ECS characteristic waves and an amplitude variability model have been developed for comprehensive analysis of morphological and rhythmic diagnostic features of cardiac signals. Experimental validation was conducted on ECS signals from patients with diagnoses: conditional norm and extrasystole. For patients with conditional norm, high stability of temporal intervals between ECS characteristic waves is observed with a mathematical expectation of 0.776 s for all wave types and minimal amplitude variability (mathematical expectation 0.00003-0.00064 mV, variance 0.00010-0.00022 mV2). In patients with extrasystole, significant cardiac rhythm irregularity was detected with a decrease in mathematical expectation to 0.503-0.504 s (by 35%) and a three-order magnitude increase in variance (to 0.011-0.012 s2) for temporal rhythm functions. The amplitude variability function demonstrated exponential growth of all statistical parameters: mathematical expectation increased to 0.070-0.452 mV (from 233 to 15067 times), variance reached extreme values of 78.44-719.20 mV2 (5-6 order magnitude increase), range varied within 46.2-122.9 mV (960 to 1500 times increase). The proposed discrete mathematical models successfully combine temporal rhythm functions considering extreme amplitude values of ECS characteristic waves with amplitude variability functions, enabling comprehensive assessment of both rhythmic and morphological ECS features. The models demonstrate high sensitivity to pathological changes in the cardiovascular system and expand the methodological foundation for developing information technology for expert analysis of morphological and rhythmic features of cardiac signals through integration with machine learning and artificial intelligence methods.

Keywords: electrocardiographic signal modeling, model, analysis, diagnostics, algorithm, cyclic discrete random process, amplitude-time characteristics, cardiac signal analysis, mathematical modeling, time rhythm function, cardiac diagnostics, amplitude variability, signal classification, artificial intelligence (AI), machine learning system (MLS), neural network.

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ІНФОРМАЦІЙНА ТЕХНОЛОГІЯ АНАЛІЗУ ЕЛЕКТРОКАРДІОСИГНАЛІВ НА ОСНОВІ МАТЕМАТИЧНИХ МОДЕЛЕЙ ЧАСОВОЇ ТА АМПЛІТУДНОЇ ВАРІАБЕЛЬНОСТІ

В роботі представлено інформаційну технологію аналізу електрокардіосигналів на основі дискретних математичних моделей з функціями часового ритму та амплітудної варіабельності характеристичних зубців P, Q, R, S, T. Розроблено дискретну математичну модель часової функції ритму з урахуванням екстремальних амплітудних значень характеристичних зубців ЕКС та модель амплітудної варіабельності для комплексного аналізу морфологічних і ритмічних діагностичних ознак кардіосигналів. Експериментальна перевірка проведена на ЕКС пацієнтів з діагнозами: умовна норма та екстрасистолія. Для пацієнтів з умовною нормою спостерігається висока стабільність часових інтервалів між характеристичними зубцями ЕКС з математичним сподіванням 0,776 с для всіх типів зубців та мінімальною амплітудною варіабельністю (математичне сподівання 0,00003-0,00064 мВ, дисперсія 0,00010-0,00022 мВ2). У пацієнтів з екстрасистолією виявлено значну нерегулярність серцевого ритму з зменшенням математичного сподівання до 0,503-0,504 с (на 35%) та зростанням дисперсії на три порядки (до 0,011-0,012 с2) для функцій часового ритму. Функція амплітудної варіабельності продемонструвала значне зростання всіх статистичних показників: математичне сподівання збільшилось до 0,070-0,452 мВ (від 233 до 15067 разів), дисперсія досягла екстремальних значень 78,44-719,20 мВ2 (збільшення на 5-6 порядків), розмах варіював у межах 46,2-122,9 мВ (збільшення від 960 до 1500 разів). Запропоновані дискретні математичні моделі успішно поєднують часові функції ритму з урахуванням екстремальних амплітудних значень характеристичних зубців ЕКС з функціями амплітудної варіабельності, дозволяючи комплексно оцінювати як ритмічні, так і морфологічні особливості ЕКС. Моделі демонструють високу чутливість до патологічних змін серцево-судинної системи та розширюють методологічну базу для розробки інформаційної технології експертного аналізу морфологічних та ритмічних ознак кардіосигналів шляхом інтеграції з методами машинного навчання та штучного інтелекту.

Ключові слова: моделювання електрокардіосигналів, модель, аналіз, діагностика, алгоритм, циклічний дискретний випадковий процес, амплітудно-часові характеристики, аналіз кардіосигналів, математичне моделювання, часова функція ритму, кардіодіагностика, амплітудна варіабельність, класифікація сигналів, штучний інтелект (AI), система машинного навчання (MLS), нейронна мережа.

Introduction

Cardiovascular diseases (CVD) remain the leading cause of mortality worldwide, necessitating a critical need for developing effective methods of early diagnosis and monitoring of cardiovascular system status.

Electrocardiography, as a non-invasive and accessible method for recording cardiac electrical activity, plays a key role in detecting a wide spectrum of cardiological pathologies. However, despite significant progress in the field of digital biomedical signal processing, existing methods of electrocardiographic signal (ECS) analysis do not fully utilize the diagnostic potential embedded in the variability of ECS amplitude characteristics. Modern automated ECS analysis systems predominantly focus on temporal parameters of cardiac rhythm or morphological features of individual cardiocycles, yet insufficient attention is paid to systematic investigation of the dynamics of amplitude values of characteristic waves from cycle to cycle. Amplitude variability of P, Q, R, S, and T waves contains important information about the functional state of the myocardium and may serve as an indicator of pathological changes that precede clinical manifestations of disease.

Integration of artificial intelligence and machine learning methods in cardiodiagnostics opens new possibilities for improving ECS classification accuracy; however, the effectiveness of these methods significantly depends on the adequacy of mathematical models used to describe signals. The absence of a comprehensive approach that would combine analysis of temporal and amplitude characteristics of ECS within a unified mathematical model hampers the development of intelligent cardiodiagnostic systems.

The aim of this study is to develop and experimentally validate a mathematical model of ECS in the form of a cyclic discrete random process with integrated temporal rhythm and amplitude variability functions. The proposed approach is directed toward creating a methodological foundation for comprehensive analysis of morphological and rhythmic ECS features, which will allow for increasing the informativeness of automated cardiac rhythm disorder diagnosis. The scientific novelty of the study lies in the proposed model of amplitude variability of ECS characteristic waves as a diagnostic feature of cardiovascular pathologies (CVP), integrated with a cyclic discrete random process model. This creates prerequisites for developing new ECS processing algorithms capable of detecting pathological changes in early stages of disease development, when traditional analysis methods prove insufficiently sensitive.

Related works

Graphical representation of ECS remains the cornerstone of cardiovascular diagnostics, providing non-invasive access to cardiac electrical activity [1, 2]. The growing global prevalence of cardiac diseases emphasizes the importance of accurate and effective ECS interpretation, which motivates research into advanced computational methods for automated analysis [3, 4]. Digital ECS processing enables the application of complex algorithms, particularly those based on machine learning, providing more accurate and objective assessment of cardiac function [1, 5]. Application of deep learning methods is becoming increasingly widespread in ECS processing. Deep learning architectures are used for automated cardiodiagnostics of ECS [6]. Work [7] investigates the use of machine learning models to enhance reliability and adaptivity of digital ECS processing procedures.

Analysis of ECS modeling, processing, and classification methods can be significantly improved through the application of cyclic discrete random processes (CDRP). This approach integrates temporal and amplitude functions to provide comprehensive ECS analysis for cardiovascular system (CVS) status diagnosis. Using discrete cardiac contractions as input data allows for more focused analysis and reduces dataset size requirements, facilitating detection of critical temporal events and improving predictive model performance [8]. The ability to analyze ECS data at discrete time moments provides advantages in detecting subtle irregularities that may be hidden when using continuous data analysis methods, which is critically important for early diagnosis and treatment of cardiovascular diseases (CVD) [9].

Integration of amplitude functions with temporal functions allows obtaining a more complete ECS representation, capturing both strength and duration of cardiac electrical activity. Digital signal analysis provides high-quality results and flexibility in storage and future analysis [10].

Application of CDRP for ECS analysis provides a powerful methodological foundation for modeling, processing, and diagnosing cardiac rhythms. This approach gains particular relevance when working with both regular and irregular cardiac cycles. Compared to traditional periodic models, the proposed approach significantly improves accuracy, reliability, and informativeness of automated ECS analysis. CDRP and their modifications, particularly cyclically correlated random processes, provide rigorous mathematical formalization of the cyclic and stochastic nature of ECS. Application of CDRP models enables more accurate modeling of multidimensional and phase structures of ECS, demonstrating advantage over traditional periodic models, especially in cases of rhythm variability [11, 12].

Models based on CRP create prerequisites for simultaneous extraction of morphological (shape-based) and rhythmic diagnostic features from ECS within a unified methodological approach. This leads to increased speed and informativeness of automated electrocardiographic data analysis [13–15]. Rhythm-adaptive statistical estimation methods are based on the invariance property of CDRP characteristics to temporal shifts. These methods ensure obtaining unbiased and consistent estimates of ECS parameters. Experimental studies demonstrate significant reduction in estimation errors compared to non-adaptive approaches, which is particularly important for signals with irregular rhythms [16].

Methods based on CDRP and cyclic spectral analysis (e.g., using GARCH models) enhance the effectiveness of cardiac pathology detection and classification, particularly arrhythmias. Generation of sensitive diagnostic features based on these methods improves machine learning classifier performance in distinguishing normal and pathological rhythms [15, 17, 18]. Using mathematical properties of CDRP, sensitivity and specificity of diagnostic systems based on ECS analysis can be increased. Such an approach allows extraction of diagnostic features sensitive to CVS changes, thereby increasing informativeness and reliability of automated diagnostic tools.

Application of CDRP in ECS analysis has the following advantages:

- CDRP have high sensitivity to CVS changes, which is crucial for early pathology detection. Estimates of mathematical expectation and cross-correlation functions when decomposed into Fourier series demonstrate significant sensitivity to cardiovascular changes, making them valuable diagnostic characteristics [15].
- Diagnostic features are served by decomposition coefficients of statistical ECS estimates obtained from CDRP. These coefficients, selected based on energy criteria, provide effective capture of cardiac signal statistical estimate energy, helping distinguish normal and pathological states. Also, using CDRP allows modeling diagnostic spaces that assist in distinguishing and grouping diagnostic features. This modeling facilitates identification of spectral coefficient groups corresponding to normal and pathological ECS models [19].
- CDRP can be integrated with advanced signal processing methods, such as wavelet transforms and deep learning models, to further enhance diagnostic accuracy. For example, it has been shown that combining cyclic processes with multiscale discrete wavelet transformations and deep neural networks effectively classifies numerous CVD [20].
- Integration of CDRP with digital signal processing (DSP) algorithms, such as Fourier spectra and wavelet spectra, supports development of automated diagnostic systems. These systems can effectively process ECS, providing valuable information for diagnosing various cardiovascular conditions [21].

Models based on CDRP and rhythm-adaptive methods are successfully integrated into specialized software for automated ECS analysis. Developed systems are capable of performing signal segmentation, rhythm assessment, and statistical processing in automatic mode. Functional capabilities of such systems include automatic detection of rhythm disorders and other pathologies, providing support for more effective and early CVD diagnosis [13–15, 22, 23].

Cyclic Discrete Random Process Model with Temporal Rhythm Function

Based on works [24, 25], key characteristics of cyclic random processes are considered, particularly the definition of rhythm function and cyclic random process with continuous parameter. The mathematical model of a cyclic signal in general form represents a random process $\xi(\omega, t),\ \omega \in \boldsymbol{\Omega},\ t \in \boldsymbol{R}$ (ξ: $\boldsymbol{R} \rightarrow L_2(\Omega, P)$), defined on probability space ($\boldsymbol{\Omega}, \boldsymbol{F}, \boldsymbol{P}$) and the set $\boldsymbol{R}$ of real numbers. Parameter t can be interpreted as spatial or temporal coordinate from a physical perspective, while the value set forms a space of random variables defined in the same probability space ($\boldsymbol{\Omega}, \boldsymbol{F}, \boldsymbol{P}$).

Work [24] provides the definition of discrete random process $\xi(\omega, t_{ml}), \omega \in \boldsymbol{\Omega}, t_{ml} \in \boldsymbol{D}$, which is classified as a cyclic discrete random process in the presence of discrete function $T(t_{ml}, n)$ that describes intervals between in-phase samples l and corresponds to rhythm function criteria. In this case, finite-dimensional vectors $(\xi(\omega, t_{m_1 l_1}), \xi(\omega, t_{m_2 l_2}), \ldots, \xi(\omega, t_{m_k l_k}))$ and $\xi\left(\omega, t_{m_1 l_1} + T(t_{m_1 l_1}, n)\right),\ \xi\left(\omega, t_{m_2 l_2} + T(, t_{m_2 l_2}, n)\right), \ldots, \xi\left(\omega, t_{m_k l_k} + T(t_{m_k l_k}, n)\right), n \in \boldsymbol{Z}$, for all integer values k ≥ 1 demonstrate stochastic equivalence in the wide sense.

Domain $\boldsymbol{D} = \{t_{ml},\ m \in \boldsymbol{Z},\ l = \overline{1, L},\ L \geq 2\}$ is the definition domain of discrete cyclic random process $\xi(\omega, t_{ml})$, where index m corresponds to the cycle number of the cyclic random process, and l denotes the sample number of the discrete random process within its m-th cycle [24].

For a discrete cyclic random process, the set of its distribution functions corresponds to the following equalities [24]:

$$F_{k_\xi}(x_1, \ldots, x_k, t_{m_1 l_1}, \ldots, t_{m_k l_k}) = F_{k_\xi}(x_1, \ldots, x_k, t_{m_1 l_1} + T(t_{m_1 l_1}, n), \ldots, t_{m_k l_k} + T(t_{m_k l_k}, n)),$$
$$x_1, \ldots, x_k \in \mathbf{R}, t_{m_1 l_1}, \ldots, t_{m_k l_k} \in \mathbf{D}, n \in \mathbf{Z}, k \in \mathbf{N}.$$

Temporal Rhythm Function Considering Extreme Amplitude Values of ECS Characteristic Waves

The temporal rhythm function effectively describes general rhythmic characteristics of ECS, but it does not differentiate temporal intervals between different types of ECS characteristic waves, which limits the possibilities of detecting local disorders of cardiac electrical activity. To enhance the diagnostic informativeness of rhythmic characteristics of individual waves, the authors developed a modified temporal rhythm function considering extreme amplitude values of ECS characteristic waves. The discrete mathematical model of the temporal rhythm function considering extreme amplitude values of ECS characteristic waves is represented by function $T_{A_k}(m)$, which accounts for extreme amplitude values of ECS characteristic waves (P, Q, R, S, and T):

$$T_{A_k}(m) = t_{A_k}(m) - t_{A_k}(m-1), \qquad k \in \{P, Q, R, S, T\}, \qquad m \in \boldsymbol{Z} \tag{1}$$

where: $t_{A_k}(m)$ – time moment of reaching the peak of k-type wave in the m-th cardiocycle (s);
$t_{A_k}(m-1)$ – time moment of reaching the peak of k-type wave in the previous cardiocycle (m-1) (s);

$T_{A_k}(m)$ – value of temporal rhythm function considering extreme amplitude values of ECS characteristic waves, reflecting the temporal interval between peaks of *k*-type waves in current *m* and previous cardiocycle (*m-1*);

$k \in \{P, Q, R, S, T\}$ – type of characteristic wave;

$m \in \boldsymbol{Z}$ – cycle numbers.

The temporal rhythm function considering extreme amplitude values of ECS characteristic waves $T_{A_k}(m)$ is characterized by the following properties:

1. Defined for all $m \geq 2$, since it requires a previous cycle for calculation.
2. Value domain: $T_{A_k}(m) \in (0, +\infty)$, s.

For quantitative description of function $T_{A_k}(m)$, a statistical processing method is used that allows calculating the following statistical parameters:

1. Estimate of mathematical expectation of temporal intervals:

$$\widehat{m}_{T_{A_k}} = \frac{1}{M}\sum_{m=1}^{M} T_{A_k}(m) = \frac{1}{M}\sum_{m=1}^{M}\left[t_{A_k}(m) - t_{A_k}(m-1)\right] \tag{2}$$

2. Estimate of variance of temporal intervals:

$$\hat{d}_{T_{A_k}} = \frac{1}{M-1}\sum_{m=1}^{M}\left[T_{A_k}(m) - \widehat{m}_{T_{A_k}}\right]^2 = \frac{1}{M-1}\sum_{m=1}^{M}\left[\left(t_{A_k}(m) - t_{A_k}(m-1)\right) - \widehat{m}_{T_{A_k}}\right]^2 \tag{3}$$

3. Range of values (variational range) of temporal intervals:

$$R_{T_{A_k}} = \max_{m=1,2,\ldots,M} T_{A_k}(m) - \min_{m=1,2,\ldots,M} T_{A_k}(m) \tag{4}$$

where: M – total number of analyzed cardiocycles;

$k \in \{P, Q, R, S, T\}$ – type of characteristic wave.

Mathematical Model of Amplitude Variability

For modeling amplitude variability of ECS waves, set ***I*** = *{P, Q, R, S, T}* is defined, which encompasses types of signal characteristic waves. For each wave type $k \in \boldsymbol{I}$ and each cycle $m \in \boldsymbol{Z}$ with discretization step *m = 1*, amplitude value $A_k(m)$ is determined, characterizing the amplitude of a specific wave in the corresponding cardiocycle.

The mathematical model of amplitude variability is represented by function $V_k(m)$, which accounts for amplitude values of ECS characteristic waves (P, Q, R, S, and T):

$$V_k(m) = A_k(m) - A_k(m-1), \quad k \in \{P, Q, R, S, T\}, \quad m \in \boldsymbol{Z}, \tag{5}$$

where: $A_k(m)$ – amplitude of *k*-type wave in the *m*-th cardiocycle (mV);

$A_k(m-1)$ – amplitude of *k*-type wave in the previous valid cardiocycle (mV);

$V_k(m)$ – value of ECS wave amplitude variability function, reflecting the amplitude change of *k*-type waves between current *m* and previous cardiocycle (*m-1*).

The amplitude variability function $V_k(m)$ is defined on set $\boldsymbol{I} \times \boldsymbol{Z}$, where ***I*** is the set of wave types {P, Q, R, S, T}, and **Z** is the set of cardiocycle indices. This function is characterized by the following properties:

1. $V_k(m)$ can take both positive and negative values, corresponding to increase or decrease in wave amplitude from cardiocycle to cardiocycle.
2. For each *k*-type wave, function $V_k(m)$ forms a sequence of values $\{\left(A_k(m) - A_k(m-1)\right), m \in \boldsymbol{Z}\}$.

For quantitative description of $V_k(m)$ and diagnostic analysis, a statistical processing method is used that allows calculating the following statistical parameters:

1. Estimate of mathematical expectation:

$$\widehat{m}_{V_k} = \frac{1}{M}\sum_{m=1}^{M} V_k(m) = \frac{1}{M}\sum_{m=1}^{M}\left[A_k(m) - A_k(m-1)\right] \tag{6}$$

2. Estimate of variance:

$$\hat{d}_{V_k} = \frac{1}{M-1}\sum_{m=1}^{M}\left[V_k(m) - \widehat{m}_{V_k}\right]^2 = \frac{1}{M-1}\sum_{m=1}^{M}\left[\left(A_k(m) - A_k(m-1)\right) - \widehat{m}_{V_k}\right]^2 \tag{7}$$

3. Range of values (variational range):

$$R_{V_k} = \underset{m=1,2,\ldots,M}{max}\{V_k(m)\} - \underset{m=1,2,\ldots,M}{min}\{V_k(m)\} \tag{8}$$

Experiments

For modeling and processing, ECS recordings from healthy patients (diagnosis: conditional norm) (Fig. 1) and a patient with diagnosed extrasystole (Fig. 2) were used.

In practical application of the discrete mathematical model of temporal rhythm function considering extreme amplitude values of ECS characteristic waves, ECS of patients with diagnoses: conditional norm and extrasystole were analyzed. Temporal rhythm functions considering extreme amplitude values of ECS characteristic waves (diagnosis: conditional norm) $T_{A_P}(m)$, $T_{A_R}(m)$, $T_{A_T}(m)$ are shown in Fig. 3.

Temporal rhythm functions considering extreme amplitude values of ECS characteristic waves (diagnosis: extrasystole) $T_{A_P}(m)$, $T_{A_R}(m)$, $T_{A_T}(m)$ are shown in Fig. 4.

Graphical realizations of amplitude variability for P wave ($V_P(m)$), R wave ($V_R(m)$), and T wave ($V_T(m)$) of patient with diagnosis: conditional norm are shown in Fig. 5, patient with diagnosis: extrasystole – in Fig. 6.

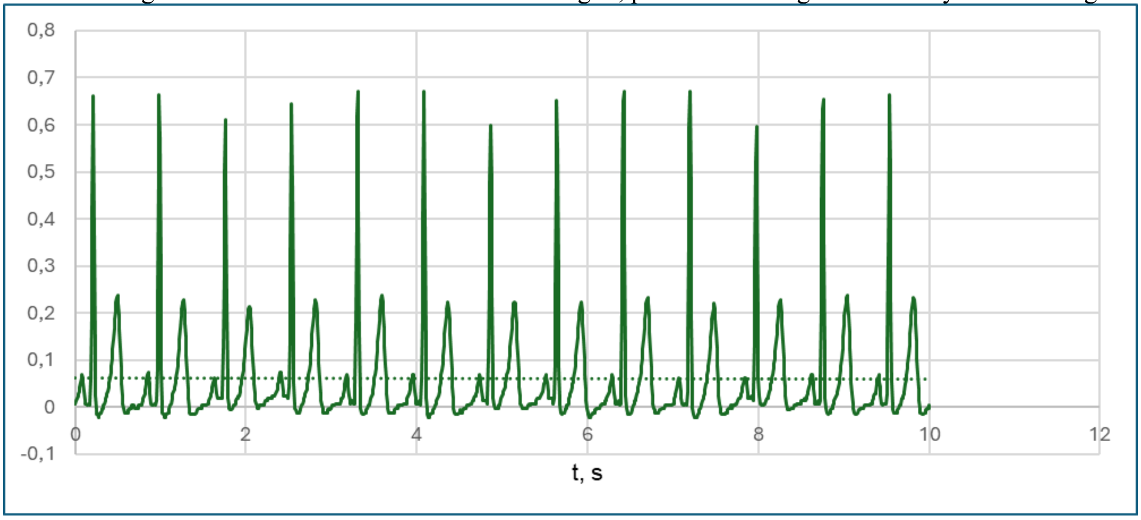


Fig. 1. ECS realization of patient (diagnosis: conditional norm)

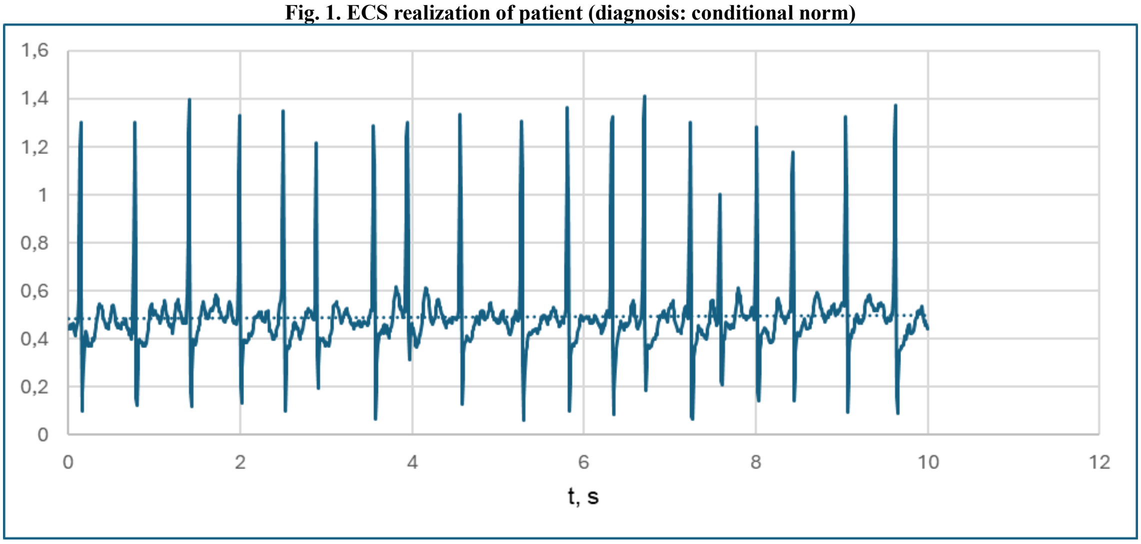


Fig. 2. ECS realization of patient (diagnosis: extrasystole)

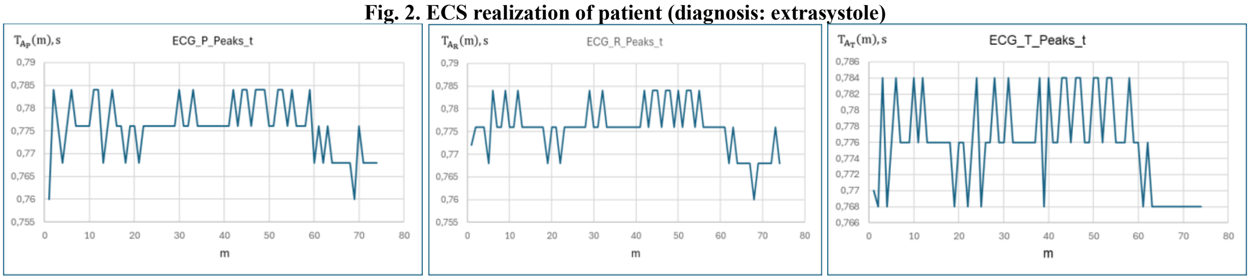


Fig. 3. Temporal rhythm functions considering extreme amplitude values of ECS characteristic waves (diagnosis: conditional norm) $T_{A_P}(m), T_{A_R}(m), T_{A_T}(m)$

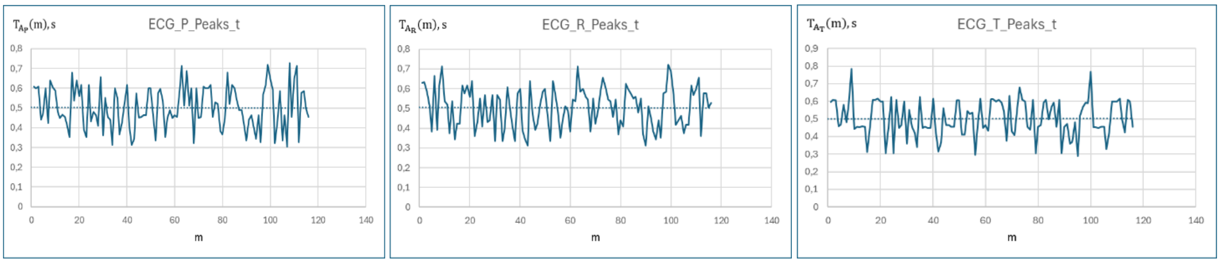


Fig. 4. Temporal rhythm functions considering extreme amplitude values of ECS characteristic waves (diagnosis: extrasystole) $T_{A_P}(m)$, $T_{A_R}(m)$, $T_{A_T}(m)$

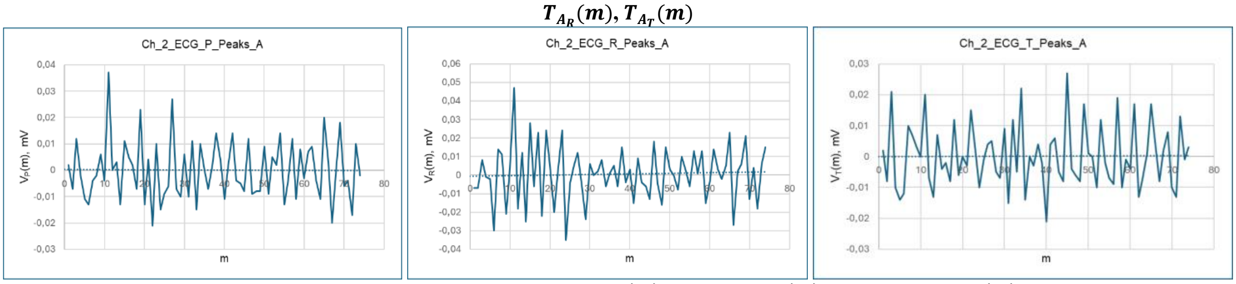


Fig. 5. Graphical realization of amplitude variability for P wave ($V_P(m)$), R wave ($V_R(m)$), and T wave ($V_T(m)$) diagnosis: conditional norm

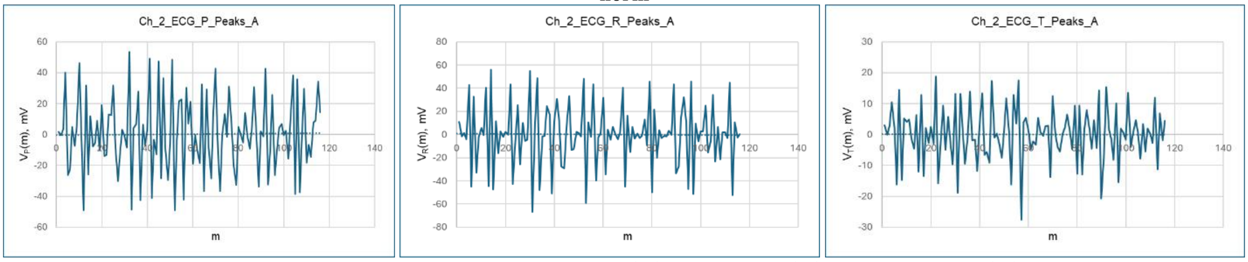


Fig. 6. Graphical realization of amplitude variability for P wave ($V_P(m)$), R wave ($V_R(m)$), and T wave ($V_T(m)$) diagnosis: extrasystole

Results of statistical processing of temporal rhythm function indicators considering extreme amplitude values of ECS characteristic waves and amplitude variability function for ECS of patient (diagnosis: conditional norm) and patient with extrasystole are presented in Tables 1–2.

Table 1

Statistical characteristics of temporal rhythm function indicators and amplitude variability function of ECS patient (diagnosis: conditional norm)

Wave type, k	Temporal rhythm function indicators			Amplitude variability function indicators		
	$\hat{m}_{T_{A_k}}, s$	$\hat{d}_{T_{A_k}}, s^2$	$R_{T_{A_k}}, s$	$\hat{m}_{V_k}$, mV	$\hat{d}_{V_k}$, mV^2	R_{V_k}, mV
P	0,776	0,00004	0,024	0,00003	0,00013	0,058
R	0,776	0,00003	0,024	0,00064	0,00022	0,082
T	0,776	0,00003	0,016	0,00015	0,00010	0,048

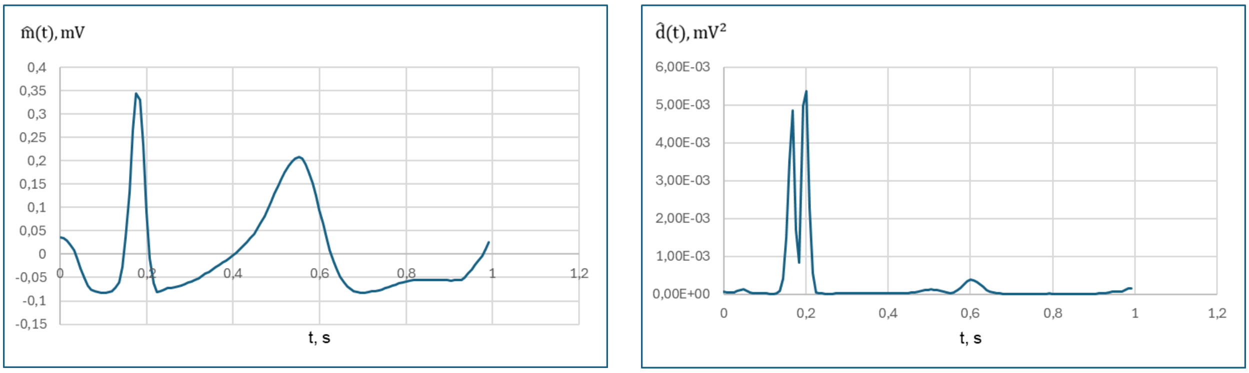
Table 2

Statistical characteristics of temporal rhythm function indicators and amplitude variability function of ECS patient with extrasystole

Wave type, k	Temporal rhythm function indicators			Amplitude variability function indicators		
	$\hat{m}_{T_{A_k}}, s$	$\hat{d}_{T_{A_k}}, s^2$	$R_{T_{A_k}}, s$	$\hat{m}_{V_k}$, mV	$\hat{d}_{V_k}$, mV^2	R_{V_k}, mV
P	0,503	0,012	0,424	0,452	576,81	102,6
R	0,504	0,011	0,408	0,070	719,20	122,9
T	0,503	0,011	0,496	0,125	78,44	46,2

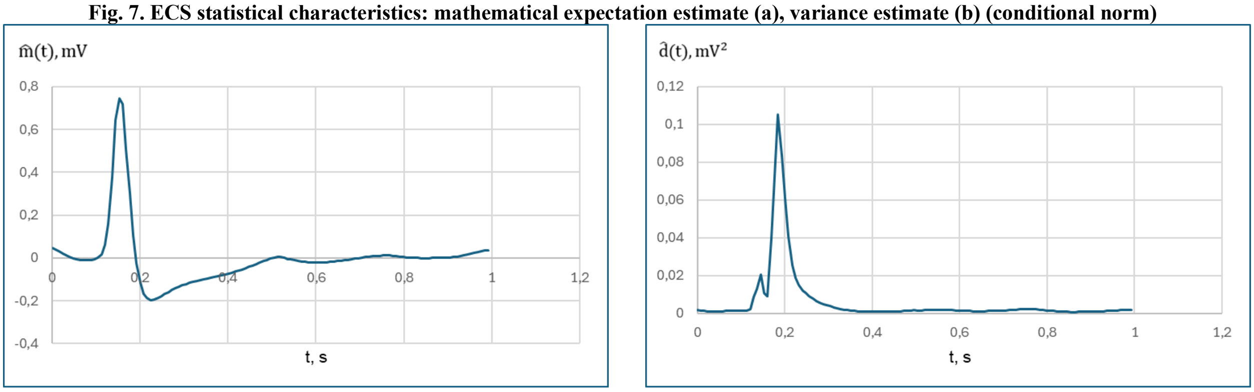
Graphical representations of ECS statistical characteristics, namely the estimate of mathematical expectation (a) and the estimate of variance (b), are shown in Figures 7–8.

Results of frequency analysis using Fourier series decomposition in the form of cosine spectrum of mathematical expectation estimate realizations (a) and sine spectrum of variance estimate realizations (b) are shown in Figures 9–10.



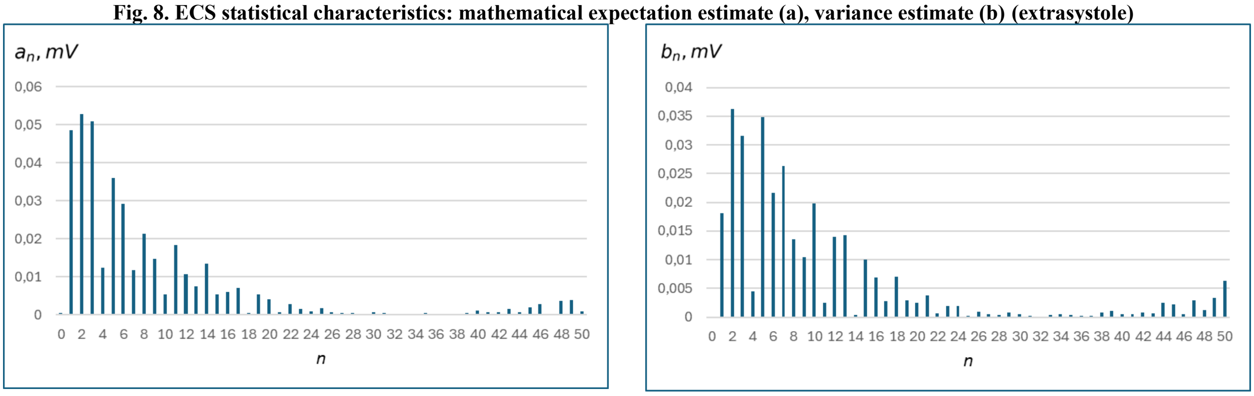
a) b)

Fig. 7. ECS statistical characteristics: mathematical expectation estimate (a), variance estimate (b) (conditional norm)



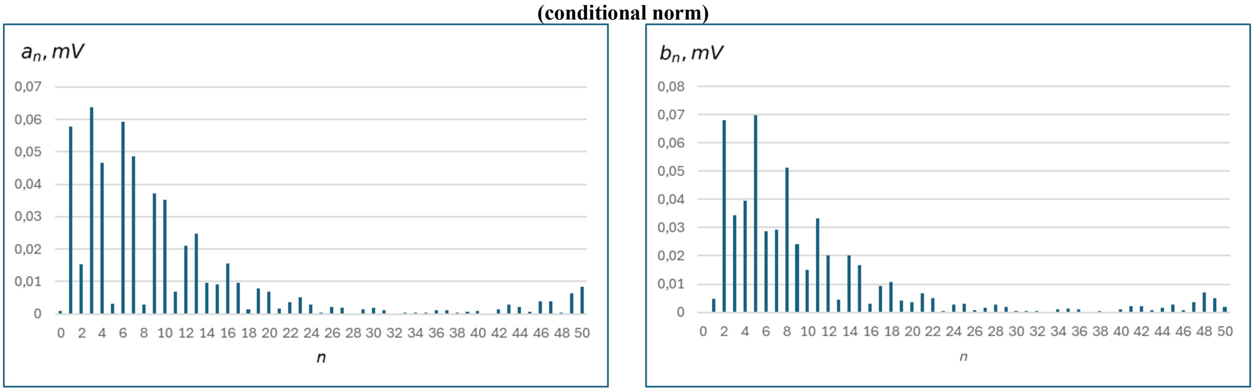
a) b)

Fig. 8. ECS statistical characteristics: mathematical expectation estimate (a), variance estimate (b) (extrasystole)



a) b)

Fig. 9. Cosine spectrum of mathematical expectation estimate realizations (a) and sine spectrum of variance estimate realizations (b) (conditional norm)



a) b)

Fig. 10. Cosine spectrum of mathematical expectation estimate realizations (a) and sine spectrum of variance estimate realizations (b) (extrasystole)

Analysis of statistical processing results of temporal rhythm function and amplitude variability function indicators presented in Tables 1 and 2 revealed significant differences between ECS characteristics of patients with diagnosis: conditional norm and extrasystole. For the patient with conditional norm (Table 1), high stability of temporal intervals between ECS characteristic waves is observed. Mathematical expectation for all wave types (P, R, T) equals 0.776 s, indicating cardiac rhythm regularity. Variance ranges within 0.00003-0.00004 s^2, and range varies from 0.016 to 0.024 s, indicating minimal variability of temporal intervals between consecutive cardiocycles. In contrast, in the patient with extrasystole (Table 2), significant cardiac rhythm irregularity was detected. Mathematical expectation of temporal rhythm function decreases to 0.503-0.504 s, which is 35% less compared to normal. Variance increases by three orders of magnitude (0.011-0.012 s^2), and range reaches 0.408-0.496 s, which is 17 to 31 times higher than corresponding normal indicators. Such changes characterize disruption of cardiac rhythm regularity, typical for extrasystole.

Amplitude variability function indicators proved particularly informative. In the patient with conditional norm, amplitude variability is characterized by minimal changes: mathematical expectation fluctuates within 0.00003-0.00064 mV, variance 0.00010-0.00022 mV^2, range 0.048-0.082 mV. This indicates stability of myocardial electrical activity throughout consecutive cardiocycles. In extrasystole, significant increase in all statistical indicators of amplitude variability is observed. Mathematical expectation increases to 0.070-0.452 mV (from 2333 to 15067 times), variance reaches extreme values of 78.44-719.20 mV^2 (5-6 order magnitude increase), and range varies within 46.2-122.9 mV (960 to 1500 times increase). The largest changes were recorded for P and R waves, which may reflect disruption of atrial and ventricular depolarization processes in extrasystole.

Despite the promising results obtained, the proposed mathematical models have several limitations that will be addressed in future studies. In particular, experimental validation was conducted on a limited dataset for conditional norm and extrasystole cases. Additionally, the ECG signals must be of high quality with clearly identifiable P, Q, R, S, and T waves.

Conclusions

The conducted study demonstrated the effectiveness of the proposed discrete mathematical models of temporal and amplitude variability for ECS analysis and classification. The developed models successfully combine temporal rhythm function considering extreme amplitude values of ECS characteristic waves with amplitude variability function, enabling comprehensive assessment of both rhythmic and morphological ECS features. Experimental validation on real patient data with diagnosis: conditional norm and extrasystole confirmed high sensitivity of models to pathological CVS changes.

The mathematical expectation of temporal intervals for patients with conditional norm remained constant at 0.776 s for all wave types with minimal variance (0.00003-0.00004 s^2). Amplitude variability maintained low values with mathematical expectation ranging from 0.00003 to 0.00064 mV and variance within 0.00010-0.00022 mV^2. Regarding temporal irregularity, a 35% decrease in mathematical expectation (0.503-0.504 s) was observed with a three-fold increase in variance (0.011-0.012 s^2) for patients with extrasystole. Statistical processing results indicate exponential growth in mathematical expectation, which increased from 233 to 15,067 times, variance increased by 5-6 orders of magnitude (from 78.44 to 719.20 mV^2), and range increased from 960 to 1,500 times for different ECG waves.

The proposed approach expands the methodological foundation for developing new algorithms for automated cardiac rhythm disorder diagnosis. Integration of amplitude and temporal characteristics creates prerequisites for improving differential CVP diagnosis accuracy. Study results open perspectives for further improvement of cardiodiagnostic information technologies through integration of the developed model with machine learning and artificial intelligence methods.

This approach also demonstrates limitations in validation scope (conditional norm versus extrasystole) and dependence on high-quality ECG signals for accurate wave identification. Future research will be directed toward expanding the experimental database for different arrhythmia types, optimizing model parameters, and developing information technology for expert analysis of morphological and rhythmic features of cardiac signals.

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