https://doi.org/10.31891/csit-2025-3-7
UDC 681.32

Nataliia VOZNA
West Ukrainian National University
Volodymyr HRYHA
Vasyl Stefanyk Precarpathian National University
Lesia MYCHUDA
Lviv Polytechnic National University
Lidiia SHTAIER
Ivano-Frankivsk National Technical University of Oil and Gas

ADVANCED METHODS OF APPLYING CODING SYSTEMS IN THE DESIGN OF DIGITAL COMPONENTS FOR CYBER-PHYSICAL SYSTEMS

This study aims to develop highly efficient digital components for cyber-physical systems capable of self-recovery, high-speed data processing, and reliable operation in real-time environments. The paper proposes new approaches to information encoding, particularly for RGB color image representation, and the modeling of neuro-like structures using the code systems of Krestenson, Rademacher, and Haar. The methodology is based on the mathematical foundations of the residue number system (RNS), modular arithmetic, and structural analysis of digital components. Hybrid models of formal neurons, perceptrons with delay lines, and wavelet neurons are employed to solve signal classification tasks. A model of signal self-recovery in a neural bundle is developed, taking into account failures of individual elements and inhibitory effects. As a result, a fault-tolerant mechanism for information transmission in bioneural structures is implemented, along with algorithms for encoding RGB pixels in the R-C and H-C code systems. These methods ensure unambiguous decoding and allow adaptive encoding in the presence of data loss or corruption. The scientific novelty lies in the integration of biological fault-tolerance principles with digital encoding methods based on RNS, providing adaptive signal recovery without the need for complete decoding. The practical significance of the research is in the potential application of the results to digital vision devices, sensor platforms, embedded systems, and high-performance processors for intelligent computing in cyber-physical environments.

Key words: cyber-physical systems, structured data, code systems, digital components, signal processing, neuron, perceptron, pixel.

Nataliia VOZNA
Західноукраїнський національний університет
Volodymyr HRYHA
Прикарпатський національний університет імені Василя Стефаника
Lesia MYCHUDA
Національний університет «Львівська політехніка»
Lidiia SHTAIER
Івано-Франківський національний технічний університет нафти і газу

ПЕРСПЕКТИВНІ МЕТОДИ ЗАСТОСУВАННЯ КОДОВИХ СИСТЕМ ПРИ ПРОЄКТУВАННІ ЦИФРОВИХ КОМПОНЕНТІВ КІБЕРФІЗИЧНИХ СИСТЕМ

Метою дослідження є розробка високоефективних цифрових компонентів для кіберфізичних систем, здатних до самовідновлення, швидкої обробки даних та надійного функціонування в умовах реального часу. У роботі запропоновано нові підходи до кодування інформації, зокрема кольорових зображень у форматі RGB, та моделювання нейроподібних структур з використанням теоретико-числових базисів Крестенсона, Радемахера та Хаара. Методика ґрунтується на математичних засадах системи залишкових класів (СЗК), модульної арифметики та структурному аналізі цифрових компонентів. Запропоновано використання гібридних моделей формальних нейронів, перцептронів із затримками та вейвлет-нейронів для реалізації задач класифікації сигналів. Розроблено модель самовідновлення сигналів у нейронному пучку з урахуванням відмов окремих елементів та ефектів гальмування. У результаті реалізовано механізм відмовостійкого передавання інформації в біонейронних структурах, сформовано алгоритми кодування пікселів у теоретико-числових базисах Радемахера-Крестенсона та Хаара-Крестенсона з однозначним декодуванням, що дає змогу адаптувати кодування до умов втрати або пошкодження даних. Наукова новизна полягає у поєднанні принципів біологічної відмовостійкості з методами цифрового кодування на основі СЗК, що забезпечує адаптивне відновлення сигналів без потреби в повному декодуванні. Практична значимість дослідження полягає в можливості впровадження результатів у пристрої цифрового зору, сенсорні платформи, елементи вбудованих систем та високопродуктивні процесори для інтелектуальних обчислень у кіберфізичному середовищі

Ключові слова: кіберфізичні системи, структуризовані дані, кодові системи, цифрові компоненти, опрацювання сигналів, нейрон, перцептрон, піксель.

Received / Стаття надійшла до редакції 14.07.2025
Accepted / Прийнята до друку 24.08.2025

Introduction

Modern computer systems operate in close interaction with the physical environment, forming multilayer platforms of cyber-physical systems (CPS) that require high performance, reliability, and the ability to adapt in real time. A pressing issue in this field is the effective integration of computing and telecommunication technologies to provide users with reliable, intelligently processed information. Particular attention is paid to the enhancement of components for physical process monitoring, sensor interfaces, pattern recognition systems, and audio-video support, which play a key role in CPS decision-making.

The current development of CPS intended for organizing measurement processes opens up opportunities for implementing computation, reliable and secure data transmission, storage, and shared use of measuring equipment and service information in the context of physical process control. In particular, the informational and functional compatibility of such components significantly increases the efficiency of human interaction with the physical environment.

The design of CPS is of special importance due to the rapid advancement of measurement theory, data encoding, processing and transmission methods, and the growing capabilities of micro- and nanotechnologies in creating high-performance software and hardware tools for computation and communication. In CPS design, the need for fault-tolerant, high-speed, and fully functional digital components highlights the relevance of using coding systems, including residue number systems (RNS), Crestenson and Rademacher systems, and neuroprocessor architectures. This work is based on the scientific school of Ya.M.Nikolaychuk [1–4], which studies models of digital structures with optimally minimal-maximal system characteristics, including specialized processors for monitoring, forecasting, and control tasks. The formalization of complexity criteria for digital components, such as hardware, time, structural, and informational complexity, enables comparative analysis of system parameters in the design of computing tools.

The properties of approximation and real-time operation of neural structures in digital signal identification processors, including models of threshold neurons, perceptrons with delay lines, and wavelet neuro-architectures, make it possible to effectively implement classification and recognition tasks in digital systems.

This paper presents the feasibility of constructing a mathematical model for signal self-recovery based on the Krestenson system. The model accounts for damage or failure of individual neurons and demonstrates how the system can adaptively reorganize while preserving the integrity of transmitted information. These principles are inspired by the structure of the bionic axon and are implemented through digital encoding with redundant moduli and inhibitory interactions in the neural bundle. Part of the paper is devoted to methods for encoding color images in the RGB format based on the Rademacher–Krestenson and Haar–Krestenson systems. An approach is proposed for representing pixels in the RNS using pairwise coprime moduli, ensuring high accuracy, unambiguous decoding, and self-recovery in the event of partial data loss.

Thus, the development of efficient digital components and next-generation processors capable of high-speed data processing, self-recovery, and robust performance under intensive informational loads in CPS remains a highly relevant and promising area of research.

Related works

One of the key challenges in the development of multi-level architectures of CPS is the effective integration of modern advances in computer and telecommunication technologies [5–7]. The wide variety of component types that form such systems necessitates the use of high-speed measurement and computation tools to ensure the required quality of operation [1–4, 6, 7].

Equally important is the task of delivering timely and high-quality information to the user, processed using intelligent and expert technologies. In this context, special attention must be paid to the modernization of components responsible for monitoring physical processes, as well as the development of tools for communication interaction [8–10]. This contributes to enabling users to make well-grounded decisions in real-time.

Moreover, personalized services create the conditions for predicting user behavior and adapting the interface interaction with CPS elements accordingly.

System characteristic criteria for digital components of cyber-physical systems

The theory of multifunctional data structuring is founded on the evaluation of **system characteristics** of digital components in computing systems, including the following criteria [11]:

- Complexity Evaluation Based on Quine’s Method. The Quine method defines complexity by calculating the total number of inputs and outputs of a microelectronic component:

$S_K = \sum_{i=1}^{n} X_i + \sum_{j=1}^{m} Y_j$, where: X_i- input, $i \in 1, n$; - output, $j \in 1, m$; n- the number of inputs to the structure, m- the number of outputs to the structure.

- Hardware Complexity. Hardware complexity is determined by counting the number of logic elements and gates that make up the device, taking into account the number of hierarchical levels and the types of components used:

one level -$A_1 = \sum_{i=1}^{n} A_i$; two levels -$A_2 = \sum_{j=1}^{m} \sum_{i=1}^{n} A_{ij}$, three levels -$A_3 = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{l} A_{ijk}$,

where: A_1, A_2, A_3- total hardware complexity at level, i, j, k- type of structure or device component at level, m, n, l- number of structures or component types used in the system.

- Temporal Complexity. Temporal complexity is estimated by determining the total signal delay through the longest chain of sequentially connected logic or functional elements between device inputs and outputs:

$\tau = \sum_{j=1}^{m} \tau_j$, where: m- number of sequentially connected elements; τ_j- signal delay in the j-th element.

- Structural Complexity. Structural complexity is applied to digital components and hardware–software tools of computing equipment, including graphical structures and multifunctional data:

$k_c = \sum_{i=1}^{n} \alpha_i P_i$, where: $P_i \in (l, P, x, d, r, h, z, b, c, i, n, a, f)$- quantitative estimates of structure elements (e.g., line, turn, intersection, touch, branching, fill, link, letter, digit, index, symbol, sign), α_i- weighting coefficients based on expert evaluations of the structural complexity of the structured data component.

- Structural Efficiency. Structure efficiency is defined as the ratio of informational complexity to structural complexity:

$k_e = K \cdot \frac{F_C}{k_c} \Rightarrow$ max, where: K– level indicator; F_C- information complexity of the device.

These criteria enable analytical estimation and comparative evaluation of system parameters for digital components in various types of computer systems, including: analog-to-digital converters (ADCs); pattern recognition and signal processing units; signal encoding and digital signal processing modules; communication protocols between functional levels of computer systems; specialized processor modules; monitoring tools for technological processes and operator interaction interfaces.

Neural-Based Signal Identification Processors

A review of scientific publications in the fields of neural networks and neurocybernetics demonstrates significant progress in the modeling and theoretical justification of the functioning of the perceptron, neuron, and neural networks [12–15]. The structure of the threshold model of a formal neuron is shown in Fig. 1 [16].

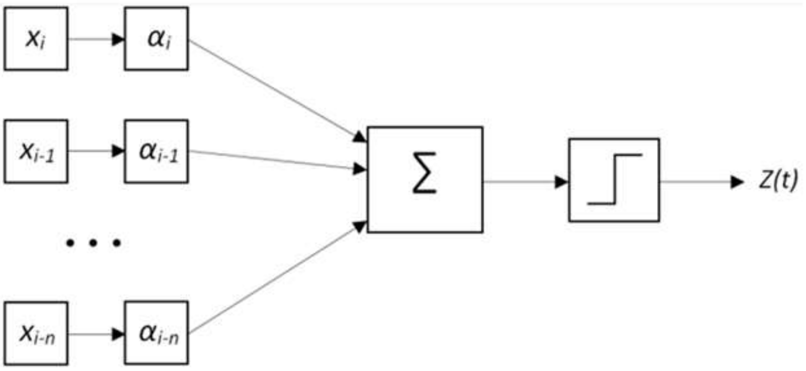

Fig 1. Structure of the threshold model of a formal neuron

The response of a formal neuron is described as follows:

$Z = f(net) = \begin{cases} 1, net > \Theta \\ 0, net \le \Theta \end{cases}$, where: $net = \sum_{i=1}^{n} x_i \cdot \alpha_i$, $-1 \le \alpha_i \le +1$, Θ- activation threshold.

To solve a wide range of problems in the field of intelligent data processing, hybrid neuro-fuzzy systems and wavelet-neuro-fuzzy architectures are increasingly being used. These systems combine the advantages of individual approaches and are characterized by improved approximation capabilities while maintaining the ability to operate in real time.

A wavelet neuron, in terms of its structure, is similar to a classical formal neuron with nn inputs. However, instead of traditional synaptic connections with weight coefficients, it uses wavelet synapses WS_i, $i = 1,2,\dots,n$. In such synapses, not only the weights w_{ji} are subject to adjustment, but also the scaling and translation parameters of the wavelet functions $\varphi ji(xi(k))$.

The analysis of signal identification processors based on threshold circuits of formal neurons demonstrates their high versatility and effectiveness when applied to a wide range of identification tasks. Adapting such architectures to the specifics of certain signal types, such as harmonic signals, enables simplification of algorithmic implementation and improvement of practical performance.

In the process of solving tasks related to prediction and recognition of components of the information description vector of an object over a given time interval $T (T = \{t_1, t_2, \dots, t_s\})$, where t - represents discrete time points at which the corresponding vector is formed, clustering and identification of the characteristic features of the studied object are carried out.

Provided that the changes in the elements of the information description vector have a deterministic nature driven by inter-parameter relationships within the cyber-physical system (CPS), it becomes possible to predict the future values of the vector. This is due to the presence of a functional dependency between previous and current parameter values, which a corresponding analytical expression can describe: $X(\omega_{t_i}) = f\left(X(\omega_{t_{i-1}}), X(\omega_{t_{i-2}}), \ldots, X(\omega_{t_0})\right)$.

The dynamic variations of individual features that constitute the information description vector can be represented as a system of partial differential equations. However, due to the absence of an explicitly known functional dependence f(°), obtaining an analytical solution to such a system is not feasible.

To overcome this complexity, the study [17] explores the possibility of using elements of artificial neural networks based on two modifications of the multilayer perceptron:

- with feedforward signal transmission;
- with feedforward signal transmission and delay lines (dynamic component of the neuron) (Fig.2).

It is evident that in the presence of simple pattern types, such as binary vectors without the use of weighting coefficients, this structure implements a threshold response of the neuron to the moving average of the input signals. When weighting coefficients are added, the neuron performs the calculation of the threshold function of the moving weighted mathematical expectation, which is functionally equivalent to the operation of digital convolution processors, as well as devices for correlation and spectral signal analysis.

Based on this class of structures, a model of a dynamic recurrent neuron oriented towards the recognition of harmonic signals was developed [18] (Fig.3).

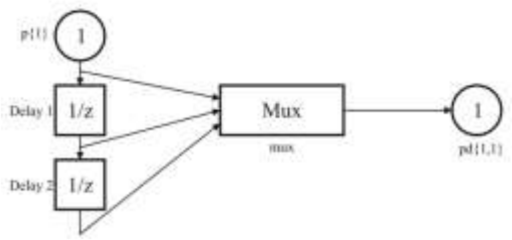

Fig. 2. Structure of a feedforward perceptron with delay lines

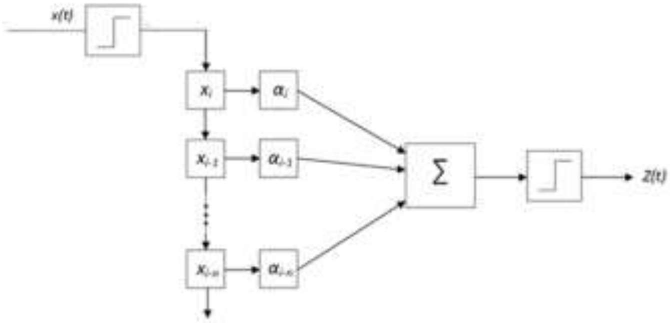

Fig. 3. Structure of a dynamic neuron for recognizing harmonic signals

The presented neuron structure includes a converter of harmonic signals into binary pattern vectors, where $x(t)$ is the input signal; x_1, x_2, … are time-delayed pattern vectors; α_1, α_2, … are weighting coefficients; and $Z(t)$ is the output signal. The model also employs threshold element circuits and a summation block.

Further research on the recurrent bioneuron, focused on pattern recognition in Hamming space, requires refinement of its structural model and formalization of functional characteristics. This defines the primary task in designing neuroprocessors for pattern recognition applications.

Data transmission method in the Krestenson residual class system in the presence of failures of individual neurons

During the study of biological objects, a distinctive feature of signal transmission in neural bundles was discovered. This phenomenon represents an effect of self-recovery and self-correction in the event of damage or death of individual groups of neurons. One of the important mathematical tasks is the development of a number-theoretic model of a self-recovering signal transmission system in bioneural fibers using the Krestenson (C) system.

It is known that neural structures can experience significant "die-off" of a large number of neurons without noticeable deterioration in the overall functional capabilities of the neural system. Similarly, when sensory information is transmitted through neural bundles, damage or failure of a substantial number of neurons practically does not affect the amplitude coding range or the dynamics of the transmitted signals. These phenomena of self-protection and self-recovery of signal transmission functions in neural bundles have prompted the development of new ideas for constructing an appropriate model of the neural bundle and theoretical justification of this phenomenon.

The creation of a mathematical model of signal self-recovery in neural fibers can be effectively applied in modern information systems to ensure reliable data transmission under conditions of intense industrial interference, signal attenuation in communication channels, or targeted unauthorized damage to transmission line components.

A unique property of the Krestenson system (CS) is the unambiguous invertibility of the forward and inverse spectral transforms while preserving phase information, which is absent in the widely used Fourier system (FS) based on harmonic functions. In the FS, phase information of the transformed signals is lost, making it impossible to fully reconstruct the signal solely using statistical and correlation characteristics after spectral analysis.

Therefore, the theoretical solution to the problem of self-recovering signal transmission in neural bundles should be based on the application of the CS. This is especially relevant considering that biological studies have unequivocally confirmed the following: in neural systems, harmonic (sinusoidal) signals are transformed into numerical pulses with variable frequency, amplitude, and periodicity. Accordingly, an adequate theoretical modeling of this problem should be carried out in the discrete form of the CS.

The solution to the problem of signal transmission in a neural bundle with a self-recovery effect is implemented as follows. Let each neuron $H_0 \div H_k$ generate an output pulse signal according to thresholds $P_1, P_2, \ldots P_k$. The input signal $x(t)$, identified in the receptor and converted into a pulse number signal $[x_i]$, is simultaneously fed to the $\oplus$ -inputs of all neurons $H_0 \div H_k$.

Within each neuron (H_j), a threshold operation is performed, described by the formula:

$$[h_j] = res\sum_{i=1}^{k}[\oplus x_i](\mathrm{mod}\, P_j),$$

where $[h_j]$ is a pulse signal whose number of pulses corresponds to the smallest non-negative remainder of the threshold operation modulo P_j, and *res* is the threshold function modulo P_j.

According to the functional structure of the neural bundle, the pulses $[h_j]$ of the j-th neuron are simultaneously fed to the inhibitory $\ominus$ -inputs of all other neurons $H_{i \neq j}$. Each neuron thus implements the threshold function in the form:

$$[h_j^*] = res\sum_{i=1}^{k}\alpha_j \cdot [h_j],$$

where α_j are the weighting coefficients of the respective neuron model.

As a result, along each fiber of the neural bundle, pulse packets $[h_j]$ are transmitted, which in the receiving neuron H_0 are again converted into the signal $[Z_i]$ according to the expression:

$$[Z_i] = res\sum_{j=1}^{k}\beta_j \cdot [h_j](\mathrm{mod}\, P_0),$$

where β_j and P_0 are the weighting coefficients and threshold modulus of the receiving neuron H_0, respectively.

It is evident that upon the death of an individual neuron H_j at the input of its axon no longer form, which removes the corresponding inhibitions h^*_j at the inputs of other neurons. This, in turn, leads to a proportional change, in particular, an increase in their activation thresholds: $P^*_j > P_j$.

Here is an example of solving the problem based on the CS, (the number theory theorem based on remainders or the chinese remainder theorem), with consideration of neural interpretation:

Let the number of neurons be $k=3$.

The system of pairwise coprime moduli: $P_1=9, P_2=10, P_3=11$.

Calculate the quantization range of the input signals $P=P_1 \cdot P_2 \cdot P_3$ $(P=9 \cdot 10 \cdot 11=990)$.

Choose the neuron activation threshold $P_0=13$.

The idea of the CS in the context of the neural model is as follows:

- each neuron responds to a signal equal to the remainder of dividing the input value by the corresponding modulus Pi;
- a neuron activates if its remainder matches a predetermined value;
- inhibition is the mutual influence among neurons where activation of one suppresses others by changing their activation thresholds or inhibiting their activity.

Let us construct the inhibition influence table (Table 1).

Table 1

Inhibition influence of each neuron on other neurons

Neuron	Inhibitory neuron	Inhibitory effect designation
H_1	H_2, H_3	$h_1^* \rightarrow H_2, H_3$
H_2	H_1, H_3	$h_2^* \rightarrow H_1, H_3$
H_3	H_1, H_2	$h_3^* \rightarrow H_1, H_2$

This corresponds to a fully connected graph with three vertices, where each neuron inhibits the other two (Fig.4).

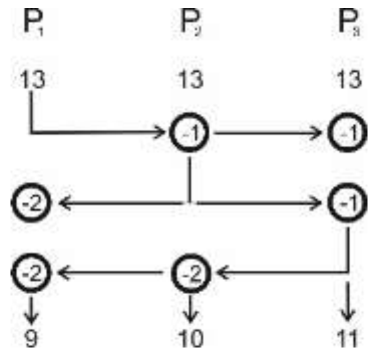

Fig.4. Graph of inhibitory influences in a neural bundle

If individual neurons fail, we get:

$$\begin{gathered} P_1=0, \quad P_2=10+1=11, \quad P_3=11+1=12; \\ P_2=0, \quad P_1=9+2=11, \quad P_3=11+1=12; \\ P_3=0, \quad P_1=9+2=11, \quad P_2=10+2=12. \end{gathered} \tag{1}$$

As a result of failures of individual neurons, the corresponding moduli disappear from the initial residue number system (RNS), and the threshold characteristics of the remaining neurons change - usually increasing due to the removal of inhibitory influences. This leads to a restructuring of the entire system, resulting in a new RNS with a reduced number of moduli corresponding to the active neurons:

$$P_i^* = \begin{cases} P_2 = 11, \quad P_3 = 12; \\ P_1 = 11, \quad P_3 = 12; \\ P_1 = 11, \quad P_2 = 12. \end{cases}$$

As a result of signal self-recovery during transmission through such a neural fiber, the signal will be unambiguous within the input signal range: $0 \le x_i \le 131$.

Let us consider an example for a specific input signal value.

Let x_i=100; P_1=9; P_2=10; P_3=11; P_0=990.

We convert the number x_i into a set of remainders b_i in the system of pairwise coprime moduli P_i. That is:

$$x_i = 100 \begin{cases} res100(mod\,9) = 1; \quad b_1 = 1; \\ res100(mod\,10) = 0; \quad b_2 = 0; \\ res100(mod\,11) = 1; \quad b_3 = 1. \end{cases}$$

The inverse transform of the RNS based on the CS is performed according to the expression:

$$N_k = res\sum_{i=1}^{k} b_i B_i (mod\, P_0\,), \tag{2}$$

Where: b_i are the smallest non-negative remainders of the number N_k modulo P_i ($0 \le b_i \le P_i - 1$);

- B_i are the orthogonal basis numbers of the RNS ($B_i = \frac{P_0}{P_i} m_i \equiv 1(mod\, P_i\,)$, where m_i are the normalizing phase coefficients of the imperfect residue transform ($0<m_i \le P_i-1$)).

Thus, the value x_i represented in the RNS code is: $x_i=100_{(10)}=(b_1, b_2, b_3)_{(9,10,11)}=(101)_{(9,10,11)}$.

Calculation of the basis numbers:

$$\begin{gathered} B_1 = 990/9 \cdot m_1 \equiv 1(\text{mod}\,9) = 110 m_1 \equiv 1(\text{mod}\,9); \quad m_1 = 5,\, B_1 = 550; \\ B_2 = 990/10 \cdot m_2 \equiv 1(\text{mod}\,10) = 99 m_2 \equiv 1(\text{mod}\,10); \quad m_2 = 9,\; B_2 = 891; \\ B_3 = 990/11 \cdot m_3 \equiv 1(\text{mod}\,11) = 90 m_3 \equiv 1(\text{mod}\,11); \quad m_3 = 6,\; B_3 = 540. \end{gathered} \tag{3}$$

Using (2) and (3), the inverse RNS transform is performed:

$$x_i = (b_1 B_1 + b_2 B_2 + b_3 B_3)(\text{mod}\,990) = (1 \cdot 550 + 0 \cdot 891 + 1 \cdot 540)(\text{mod}\,990) = 1090\,(\text{mod}\,990) = 100.$$

Suppose neuron H_1 with modulus $P=9$ fails, i.e., $P_1=0$. Then, according to (1) and Fig.4, we obtain a new RNS: $P_2=11$, $P_3=12$, a $P_0^*=132$.

As a result of the changes in the threshold values of neurons H_2 and H_3, the information transmission according to the model will occur in the new RNS with moduli $P_2=11$ and $P_3=12$, with basis numbers calculated similarly:

$$x_i = 100 \begin{cases} res100(\mathrm{mod}11) = 1; & b_1 = 1; \\ res100(\mathrm{mod}12) = 4. & b_2 = 4. \end{cases}$$

$x_i = 100_{(10)} = (b1, b2)_{(11,12)} = (14)_{(11,12)}$;

$$B_1 = \frac{132}{11} m_1 \equiv 1(mod\,11) = 12m_1 \equiv 1(mod\,11); \quad m_1 = 1; \quad B_1 = 12;$$

$$B_2 = \frac{132}{12} m_2 \equiv 1(\mathrm{mod}12) = 11m_2 \equiv 1(\mathrm{mod}12); \quad m_1 = 11; \quad B_1 = 121.$$

Correspondingly, the inverse transform of the new RNS is performed as:

$$x_i = (1 \cdot 12 + 4 \cdot 121)(mod\,132) == 496(mod\,132) = 100.$$

Similar calculations are performed in the event of failures of H_2 and H_3.

Methods for encoding pixels of color images using theoretical and numerical representations in extended Galois fields

1. RGB pixel encoding method in Rademacher (R) and Krestenson (C) systems.

The encoding of color pixels in the Hamming space of a screen, defined in a Cartesian coordinate system, can be uniquely represented in the RNS according to the CS. This representation is achieved by selecting three pairwise coprime moduli, which enable unambiguous encoding of each pixel in the RGB system in binary form according to the RS, by performing a direct integer transformation within the RNS (2).

Unambiguous encoding of RGB pixels in the R-CS is performed by selecting appropriate residue coding range values b_i within the RS [19]:

$$b_1 = b_R; \qquad 0 \le b_R \le 255; (00000000 \div 11111111);$$

$$b_2 = b_G; \qquad 0 \le b_G \le 255; (00000000 \div 11111111);$$

$$b_3 = b_B; \qquad 0 \le b_B \le 255; (00000000 \div 11111111).$$

Taking into account the coefficients $m = 1.0$, $n = 4.5907$, $p = 0.0601$, the permissible value range for the most saturated shade of green can be defined within the interval $0 \le b_G \le 254$, which ensures the pairwise coprimality of the moduli $P_1 = 256$, $P_2 = 255$, $P_3 = 257$.

Let us verify that the selected system of moduli satisfies the mutual coprimality condition by factoring them: $256 = 2^8$, $255 = 5 * 51$, 257 is a prime number.

That is, $P_0 = 16776960$, where $P_0 < 2^{24} = 16777216$.

Thus, the condition for forming a 24-bit pixel code in the R-CS is fulfilled.

In the binary number system, the RS codes of the moduli are represented as:

$P_1 = 100000000_{(2)}; P_2 = 11111111_{(2)}; P_3 = 100000001_{(2)}$.

Therefore: $P_0 = 111111111111111100000001_{(2)}$

Since the set of moduli P_1, P_2, P_3 includes the modulus$P_1 = 2^8$, the residue of a number N_k (corresponding to the green color component, G) according to the inverse transformation in the RNS can be obtained without the need for decoding - it is directly represented as the 8 least significant bits of the number N_k, expressed in the RS.

By solving equation (3), we obtain the values of the multiplicative inverses m_i and the basis numbers B_i:

$$m_1 = 255, \ B_1 = 16711425;$$

$$m_2 = 128, \ B_2 = 8421376;$$

$$m_3 = 129, \ B_3 = 8421120.$$

Let us verify the accuracy of the calculations:

$$N_k = (b_R \cdot B_1 + b_G \cdot B_2 + b_B \cdot B_3) \cdot (\mathrm{mod}\,P_0) = 1 \text{ at } b_R = 1, \ b_G = 1, \ b_B = 1.$$

That is $N_k = (1 \cdot 16711425 + 1 \cdot 8421376 + 1 \cdot 8421120) \cdot (\mathrm{mod}\,P_0) = 1$.

Let, for example: $R = 10$, $G = 200$, $B = 100$.

That is $N_k = (10 \cdot 16711425 + 200 \cdot 8421376 + 100 \cdot 8421120) \cdot (\mathrm{mod}16776960) = 9187850$.

The obtained value corresponds to the binary representation of an RGB pixel in the CS ($100011000011001000001010_2$).

The result of decoding such a representation of an RGB pixel is:

$$r_i = resN_k(\bmod P_1);\ g_i = resN_k(\bmod P_2);\ b_i = resN_k(\bmod P_3).$$

2. Method of encoding RGB standard pixels in the Rademacher–Krestenson (R–C) and Haar–Krestenson (H–C) systems.

The encoding of color image pixels in the RGB format is performed using a 24-bit binary representation, in which the intensities of the red, green, and blue channels are specified separately as 8-bit binary codes formed in the RS.

$$R\begin{cases} r_{8-1} \\ \ldots \\ r_i \\ \ldots \\ r_0 \end{cases};\qquad G\begin{cases} g_{8-1} \\ \ldots \\ g_i \\ \ldots \\ g_0 \end{cases};\qquad B\begin{cases} b_{8-1} \\ \ldots \\ b_i \\ \ldots \\ b_0 \end{cases};$$

$$0 \le r_i \le 255;\qquad 0 \le g_i \le 255;\qquad 0 \le b_i \le 255.$$

The encoding of RGB pixels of color images in the R-C or H-C systems is carried out by selecting a set of pairwise coprime moduli (P_1, P_2, P_3), whose product exceeds the full quantization range of brightness (r_i, g_i, b_i). This requirement can be satisfied using various combinations of moduli within the framework of the discrete RNS transformation. For example, a set of moduli $P_1 = 5, P_2 = 7, P_3 = 8$ enables unambiguous encoding of the brightness components r_i, g_i and b_i within the range $P_0 = 5*7*8 = 280 > 255$.

As a result, a corresponding code structure is formed in the R–CS, providing an unambiguous representation of each RGB pixel as a residue code:

$$R \vee G \vee B\begin{cases} a_2 \\ a_1 \\ a_0 \end{cases};\qquad \begin{cases} c_2 \\ c_1 \\ c_0 \end{cases};\qquad \begin{cases} d_2 \\ d_1 \\ d_0 \end{cases};$$

$$P_1 = 5;\qquad P_2 = 7;\qquad P_3 = 8,$$

where: $a_i \in \overline{0,1}$; $c_i \in \overline{0,1}$; $d_i \in \overline{0,1}$; $i \in \overline{0,2}$.

The code structure of an RGB pixel in the R-C system with the selected set of moduli $P_1 = 5, P_2 = 7, P_3 = 8$ will have the form shown in Table 2/

Table 2

Code structure of an RGB pixel in the Rademacher–Krestenson system

Color component	Value (0–255)	The remainder of P_1=5	The remainder of P2=7	The remainder of P3=8	Presentation in R–C system
r (Red)	R	r_1=*R* mod 5	r_2=R mod 7	r_3=R mod 8	(r_1, r_2, r_3)
g (Green)	G	g_1=G mod 5	g_2=G mod 7	g_3=G mod 8	(g_1, g_2, g_3)
b (Blue)	B	b_1=B mod 5	b_2=B mod 7	b_3=B mod 8	(b_1, b_2, b_3)

The values a_i, c_i, d_i are calculated as remainders: $a_i = res(r_i \bmod P_1)$, $c_i = res(g_i \bmod P_2)$, $d_i = res(b_i \bmod P_3)$.

Based on the solution of the Diophantine equations (3), we obtain the multiplicative inverses m_i and the basis numbers B_i for the given set of moduli:

$$m_1 = 1;\ B_1 = 56;\ m_2 = 3;\ B_2 = 120;\ m_3 = 3;\ B_3 = 105. \quad (4)$$

Let us verify the correctness of the obtained values m_i and B_i according to expression (2):

$$N_1 = (1 \cdot 56 + 1 \cdot 120 + 1 \cdot 105) \bmod 280 = 1$$

For example, let the intensity values of the RGB pixel be given as: $r_i = 10$; $g_i = 100$; $b_i = 37$

We then obtain the RGB pixel codes as follows:

1. In the RS: $r_i = 00001010_{(2)}$; $g_i = 01100100_{(2)}$; $b_i = 00100101_{(2)}$.

2. In the R-CS: $r_i = (\overbrace{000}^{P_1}\overbrace{011}^{P_2}\overbrace{101}^{P_3})_{(5,7,8)}$; $g_i = (\overbrace{000}^{P_1}\overbrace{010}^{P_2}\overbrace{010}^{P_3})_{(5,7,8)}$; $b_i = (\overbrace{010}^{P_1}\overbrace{010}^{P_2}\overbrace{101}^{P_3})_{(5,7,8)}$.

The representation of RGB pixel codes in the H-C system for each intensity value r_i, g_i and b_i is performed as follows:

$$R \vee G \vee B \begin{cases} a_{P_1-1} \\ .... \\ a_i \\ .... \\ a_0 \end{cases}; \qquad \begin{cases} c_{P_2-1} \\ ... \\ c_i \\ ... \\ c_0 \end{cases}; \qquad \begin{cases} d_{P_3-1} \\ ... \\ d_i \\ ... \\ d_0 \end{cases};$$

$P_1 = 5$; $P_2 = 7$; $P_3 = 8$, where $i \in \overline{0, P_i - 1}$.

For the given intensity values of the RGB pixel: $r_i = 10$; $g_i = 100$; $b_i = 37$ we obtain the corresponding code structure in the H–CS as follows:

$$r_i = (10000..0001000..00100000);$$
$$g_i = (10000..0010000..00100000);$$
$$b_i = (00100..0010000..00000100).$$

The representation of the digital values r_i, g_i and b_i in different code systems results in different bit-lengths of the corresponding code structures:

1. In the RS: $K_R = \log_2 2^8 = 8$ bit.

2. In the R-CS: $K_{R-C} = \sum_{i-1}^{3} [\hat{E}(\log_2 P_i - 1)] = 3 + 3 + 3 = 9$ bit.

3. In the H-CS: $K_{H-C} = \sum_{i=1}^{n} P_i = 5 + 7 + 8 = 20$ bit.

The method of encoding RGB pixels in the Krestenson system makes it possible to replace the individual vectors P_1, P_2, P_3 with a single vector P_0, whose bit length is 24 bits, equal to the bit depth of RGB system codes. This allows RGB pixels to be represented in Hamming space. The representation of digital data in the Rademacher–Krestenson and especially the Haar–Krestenson systems, which are based on the mathematical principles of modular arithmetic and residue number systems, enables a 2–3 order of magnitude increase in algorithm execution speed, regardless of the bit-width of the numbers.

These operations are performed during transformations for various standards of color formation, digital television, display printing, display types, modems, printers, and other devices.

Conclusions

The article examines theoretical and applied aspects of designing digital components of computer systems aimed at high-speed information processing, self-recovery, and operation within a cyber-physical environment. The proposed models and methods are based on the mathematical foundations of the RNS, particularly the Krestenson system, which ensures reliable data transmission and structural resilience of digital systems against the failure of individual elements.

Criteria for structural complexity of digital components are substantiated, allowing for the formalized comparative analysis of architectures of specialized computing devices. The effectiveness of neuron-like structures for signal recognition is demonstrated, based on perceptron and wavelet-neuron models, along with their adaptability to real-time processing.

Special attention is given to a model of signal self-recovery in a neural fiber, where redundancy of moduli and inhibitory interactions enable the implementation of biologically inspired fault tolerance. This model makes it possible to design fault-tolerant neuroprocessors for classification and control tasks in conditions of damage or partial loss of computational elements.

Efficient methods for encoding RGB pixels in the Rademacher–Krestenson and Haar–Krestenson systems ensure compact representation, high computation speed, and preservation of data integrity in discrete transformations.

The obtained results establish a scientific foundation for the development of intelligent digital components and special-purpose processors within computer systems and elements of cyber-physical platforms focused on sensors, embedded systems, wireless communication, and interference-resistant data encoding.

Thus, the integration of code system properties, neuron-like architectures, and RNS encoding opens new prospects for designing high-performance, adaptive, and reliable components of computer systems.

References

1. Specialized Computer Technologies in Informatics: Monograph edited by Y. M. Nykolaychuk, Ternopil: Beskydy, 2017. 919 p.

2. High-performance coprocessors for arithmetic and logic operations of multi-bit cores for vector and scalar supercomputers / Nyckolaychuk, V. Hryha, N. Vozna, A. Voronych, A. Segin, P. Humennyi // Advanced Computer Information Technologies. 12th International Conference. ACIT 2022. – Spišská Kapitula, Slovakia, September 2022. – P. 410-414.

3. High-performance multi-bit adder-accumulators as component of the ALU in supercomputers / Y. Nyckolaychuk, V. Hryha, N. Vozna, I. Pitukh and L. Hryha // publ. on CEUR Workshop Proceedings, 2023, P. 649-661.

4. Microelectronics Structures of Arithmetic Logic Unit Components / Y. Nyckolaychuk, N. Vozna, A. Davletova, I. Pitukh, O. Zastavnyy, V. Hryha // Advanced Computer Information Technologies. International Conference. ACIT'2021. – Deggendorf, Germany, September 2021. – P. 682-685.

5. A.O. Melnyk. "Cyber-physical systems multilayer platform and research framework. Advances in cyber-physical systems", Lviv Polytechnic National University Publishing, 2016, № 1 (1), p.1-6.

6. Xia Guanxiang The model of the system for objects recognition in the real-time video stream/ Xia Guanxiang, Viacheslav Kovtun // Computer Systems and Information Technologies, 2024. - №4. P.157–165.

7. James Reinders and Jim Jeffers, High Performance Parallelism Pearls Volume One: Multicore and Many-core Programming Approaches, 2014, 600p.

8. Chang C-Y, Zeadally S, Chen T-S. Cyber Physical Systems Using Sensor Technologies. International Journal of Distributed Sensor Networks. 2012;8(10). doi:10.1155/2012/957396.

9. Bielik V. Sensors in cyber-physical systems based on android operating system / Valerii Bielik, Yurii Morozov, Mykola Morozov // Advances in Cyber-Physical Systems. – Lviv : Lviv Politechnic Publishing House, 2021. – Volume 6, № 2. – P. 83–89.

10. Rana, M.M. and Bo, R. (2020), IoT-based cyber-physical communication architecture: challenges and research directions. IET Cyber-Physical Systems: Theory & Applications, 5: 25-30. https://doi.org/10.1049/iet-cps.2019.0028

11. Vozna, Nataliia. New Approach to Assessing Structural Complexity. In: Artificial Intelligence, Medical Engineering and Education. IOS Press, 2024. p. 351-365.

12. Gabrié M. et al. Neural networks: From the perceptron to deep nets //Spin Glass Theory and Far Beyond: Replica Symmetry Breaking After 40 Years. – 2023. – С. 477-497.

13. Subbotin S. O. Neural networks: theory and practice: a textbook / S. O. Subbotin. – Zhytomyr: Publ. O. O. Evenok, 2020. – 184 p.

14. Dmytrienko V. D. Neural networks: from the simplest models of biological systems to artificial intelligence systems [Electronic resource]: a textbook / Dmytrienko V. D., Leonov S. Yu., Zakovorotny O. Yu. ; National Technical University "Kharkiv Polytechnic Institute". – Electronic text data. – Kharkiv, 2025. – 232 p.

15. Müller B., Reinhardt J., Strickland M. T. Neural networks: an introduction. – Springer Science & Business Media, 2012.

16. Self-repairing signal transmission system of the bioneuron axon model in the Krestenson basis / Zavediuk T.O., Nikolaychuk Ya.M., Voronych A.R.// Bulletin of Khmelnytskyi National University, Khmelnytskyi 2012, No. 4 P.137-142.

17. Pomorova O.V. Theoretical foundations, methods and means of intellectual diagnostics of computer systems: Monograph. – Khmelnytskyi: Triada-M, 2007. – 252p.

18. Zavediuk T.O. Methods of processing harmonic signals based on special processors with neuro-like components / T.O. Zavediuk // Bulletin of the National University "Lviv Polytechnic" "Computer Sciences and Information Technologies". – Lviv. – 2013. – No. 751. – P. 18-28.

19. N.Y. Vozna Algorithms for Solving Problems of Cryptographic Protection of Color Image Pixels in the Rademacher's Basis and Residue Number Systems [Electronic resourse] / N.Y. Vozna, Y.M. Nykolaichuk,O.I. Volynskyi // Cybernetics and Systems Analysis. Springer. – Electronic data. - May 2019, Volume 55, Issue 3, pp 474–487.

Nataliia Vozna **Наталія Возна**	DrS on Engineering, Professor of Specialized Computer System Department, West Ukrainian National University, Ternopil, Ukraine e-mail: nvozna@ukr.net https://orcid.org/0000-0002-8856-1720	доктор технічних наук, професор кафедри спеціалізованих комп'ютерних систем, Західноукраїнський національний університет, Тернопіль, Україна
Volodymyr Hryha **Володимир Грига**	PhD, Associate Professor of Computer Engineering and Electronics Department, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine e-mail: volodymyr.gryga@pnu.edu.ua https://orcid.org/0000-0001-5458-525X	кандидат технічних наук, доцент кафедри комп'ютерної інженерії та електроніки, Прикарпатський національний університет імені Василя Стефаника, Івано-Франківськ, Україна
Lesya Mychuda **Леся Мичуда**	DrS on Engineering, Professor of the Department of Information Technology Security, Lviv Polytechnic National University, Lviv, Ukraine e-mail: lesia.z.mychuda@lpnu.ua https://orcid.org/0000-0001-8266-1782	доктор технічних наук, професор кафедри безпеки інформаційних технологій, Національний університет «Львівська політехніка», Львів, Україна
Lidiya Shtaiyer **Лідія Штаєр**	PhD, Associate Professor of the Department of Information and Telecommunication Technologies and Systems, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine e-mail: lidiia.shtaier@nung.edu.ua https://orcid.org/0000-0003-1013-9869	кандидат технічних наук, доцент кафедри інформаційно-телекомунікаційних технологій та систем, Івано-Франківський національний технічний університет нафти і газу, Івано-Франківськ, Україна