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CROP YIELD MODEL BASED ON MAXIMUM VALUES OF CUMULATIVE VEGETATION INDICES

This research develops a precision modeling approach for cereal crop yield estimation utilizing remote sensing data within a information architecture framework. A two-tier model is proposed wherein the first tier conducts vegetation index dynamics modeling (NDVI, MTCI) through an adaptive modified Monod model based on contemporary differential equation systems, while the second tier performs yield prediction via linear regression and machine learning methodologies to accommodate nonlinear interdependencies. An efficient parametric identification algorithm for models is developed, accounting for their nonlinearity characteristics and employing the Levenberg-Marquardt gradient method for refined parameter optimization.

An adaptive prediction algorithm based on observation window methodology is implemented, leveraging an ensemble of previously observed trajectories to maximize forecasting precision. Practical applicability is validated through numerical experiments on empirical vegetation index data from rice cultivation. The synthesized findings demonstrate the potential of the proposed methodology for addressing contemporary precision agriculture challenges, systematic food security monitoring, and strategic decision-making processes in the agricultural sector.

Keywords: vegetation indices, yield modeling, Monod model, remote sensing, machine learning

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МОДЕЛЬ УРОЖАЙНОСТІ СІЛЬСЬКОГОСПОДАРСЬКИХ КУЛЬТУР НА ОСНОВІ МАКСИМАЛЬНИХ ЗНАЧЕНЬ КУМУЛЯТИВНИХ ВЕГЕТАЦІЙНИХ ІНДЕКСІВ

У дослідженні запропоновано прецизійний підхід до математичного моделювання та прогнозування врожайності зернових культур із використанням даних дистанційного зондування Землі в межах сучасної архітектури інформаційної системи підтримки прийняття рішень. Методологічну основу становить дворівнева модель, яка забезпечує комплексне врахування динамічних і статистичних характеристик процесів росту рослин. На першому рівні здійснюється моделювання часової еволюції вегетаційних індексів NDVI та MTCI шляхом застосування адаптивної модифікованої моделі Моно, сформованої на базі систем нелінійних диференціальних рівнянь. Такий підхід дає змогу адекватно описувати фізіологічні особливості розвитку рослин та реакцію агроекосистем на змінні умови середовища.

Другий рівень моделі орієнтований на прогнозування врожайності та поєднує класичні методи лінійної регресії з інструментарієм машинного навчання, що дозволяє враховувати складні нелінійні взаємозалежності між вегетаційними індексами та кінцевими показниками продуктивності. Для забезпечення високої точності моделювання розроблено ефективний алгоритм параметричної ідентифікації, який враховує ступінь нелінійності моделей і базується на застосуванні градієнтного методу Левенберга–Марквардта для уточненої оптимізації параметрів.

Окрему увагу приділено реалізації адаптивного алгоритму прогнозування на основі методології ковзних вікон спостереження, що використовує ансамбль раніше зафіксованих траєкторій розвитку вегетаційних індексів з метою підвищення стабільності та точності прогнозів. Практичну ефективність запропонованого підходу підтверджено результатами чисельних експериментів на емпіричних даних дистанційного моніторингу посівів рису. Отримані результати свідчать про значний потенціал розробленої методології для вирішення актуальних завдань точного землеробства, оперативного моніторингу продовольчої безпеки та підтримки стратегічного управління в аграрному секторі.

Ключові слова: вегетаційні індекси, моделювання врожайності, модель Моно, дистанційне зондування, машинне навчання

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Introduction

Remote sensing represents a powerful technology for non-destructive monitoring of agricultural crop conditions [1,2]. Effective utilization of acquired data requires their structured organization in geographic information systems (GIS), which is critically important for transforming primary information into practically useful agronomic solutions.

The foundation of working with vegetation indices (NDVI, NDRE) lies in their geospatial referencing. Data are stored in the form of raster or vector layers, which are integrated with additional geospatial information—cadastral maps, meteorological data, etc. For processing large information arrays, specialized spatial databases are employed, ensuring the capability to perform complex spatial queries and analytical operations.

A key aspect of systematization is the formation of time series that demonstrate the evolution of vegetation parameters for specific territorial units. This allows for identifying vegetation development trends and timely identification of deviations from normal parameters. Classification and segmentation methods of images facilitate the identification of zones with different vegetation cover characteristics and localization of problematic areas. A comprehensive approach involves combining satellite data with meteorological, agrochemical, and cadastral information for thorough analysis of factors influencing crop conditions. The use of open-source software, particularly QGIS, significantly simplifies the processes of automation in remote sensing data processing and analysis.

Vegetation indices based on spectral reflectance coefficients have become an integral component of agricultural system modeling. Portable optical sensors mounted on unmanned aerial vehicles provide high-precision data for crop productivity forecasting [3, 4, 5].

Research [6] was aimed at identifying the optimal vegetation index for assessing plant response to elevated temperatures, heat stress, and herbicide damage. During 2016-2018, monitoring of spectral reflectance characteristics, yield components, and growth parameters (plant height, leaf area index LAI, above-ground dry biomass) of rice was conducted under controlled conditions of a field temperature chamber. Analysis of relationships between vegetation indices and productivity indicators under stress conditions showed that NDVI, MTCI, and cumulative growing degree-days form sigmoidal dependencies with high determination coefficient values under normal growth conditions. However, herbicide damage significantly reduced the amplitude of these curves. Particularly, NDVI and MTCI proved to be sensitive indicators of growth and development retardation caused by stress factors through their correlation with cumulative growing degree-days. Maximum values of these indices are traditionally used as yield predictors.

The obtained results emphasize the critical importance of predicting NDVI and MTCI dynamics both for early detection of plant stress conditions and for using their peak values in yield forecasting models.

However, modeling the accumulation of vegetation index values using logistic regression dependencies and Monod differential equation systems is complicated due to the weak predictive properties of the mentioned models. Meanwhile, Monod differential equation systems require fewer parameters for their identification and provide higher modeling accuracy, as demonstrated in previous works by the authors using the NDVI index [12,13]. By weak predictive properties of the model, we mean the complexity in predicting a single trajectory of vegetation index dynamics by observing this dynamics in early stages. This complexity can be circumvented by applying adaptive models that are built using a certain ensemble of previously observed trajectories of index variability. The predictive interval is divided into certain sections containing small volumes of observation points. Based on index values in the previous section, the nearest previously observed trajectories to the current realization are determined. A linear combination of model values of these indicators is used to predict values of the current realization for the next period. Such an adaptive model is described and applied in modeling real NDVI index trajectory values in work [13].

Vegetation indices can serve as a basis for modeling grain crop yield. The NDVI index, which is recorded using the simplest equipment, correlates well with crop green mass. At the same time, the MTCI index correlates well with chlorophyll content in plants, which greatly influences grain maturity formation. Therefore, this work generalizes the methodology for building an adaptive NDVI index model to modeling MTCI index values with subsequent rice yield modeling based on models of the mentioned indices.

Literature Review

Research [8] emphasizes the complexity of determining Monod model parameters based on experimental data, to overcome which the use of experimental design methodology is proposed. Study [9] presents explicit and implicit schemes for solving the Monod differential equation system, applying asymptotic representation to eliminate the stiffness problem when microorganism concentration approaches zero.

The authors of work [10] consider a simplified version of the Monod model, focused exclusively on microorganism dynamics, and demonstrate the possibility of its transformation into a linear regression model, which simplifies the parameter identification process. In study [11], bioreactor processes are analyzed using the full Monod model, where Matlab tools are applied for solution construction, however parameter identification issues remain outside the scope of attention.

A notable contribution to the development of vegetation index dynamics modeling methodology based on remote sensing data was made in the work of Pasichnyk et al. [12]. The authors proposed using the Monod differential equation system for NDVI dynamics modeling, which allows achieving more accurate plant development forecasting under both normal and stress conditions. The research includes detailed analysis of structural and parametric identification of the Monod model considering the complexity of nonlinear parameter estimation in differential equation systems. A specialized method for parameter identification is proposed, which accounts for model nonlinearity and uses a combination of non-uniform and uniform grids for efficient parameter space exploration. Application of the Levenberg-Marquardt gradient method for refining initial parameter estimates allows achieving high accuracy in vegetation index dynamics modeling.

Parallel development occurs in the direction of automating image annotation processes for computer vision systems in agriculture. In the work of Babala et al. [13], methods for creating image datasets and tuning classification model parameters using neural networks based on the TensorFlow framework are investigated. The scientific novelty of the work lies in developing new approaches to automated collection of thematic image collections and formalizing the methodology of parametric training for classification models. The practical significance of the research is expressed in improving the efficiency of image annotation processes for geographic information systems in the agricultural sector. The dependence of classification accuracy on training sample size and image augmentation parameters was experimentally established. The study showed that with optimal choice of augmentation parameters and using 48 images per label in the training sample, it is possible to reduce classification error to an acceptable level of 8%.

Despite numerous studies and recent achievements in the field of vegetation index modeling, problems of comprehensive integration of mathematical modeling with remote sensing data in agricultural monitoring require further development. The issue of developing universal methodologies that combine the advantages of Monod models with the capabilities of modern image processing technologies and geographic information systems remains particularly relevant. This gap creates a promising direction for further research in the field of precision agriculture.

Adaptive Model of Vegetation Index Dynamics

In the developed model of vegetation index evolution relative to the cumulative GDD indicator, the principle of irreversibility of the vegetation characteristic value accumulation process is considered, which determines the adoption of the vegetation index degradation coefficient at zero level.

Denoting the current vegetation index value as X, the cumulative GDD indicator as t, and introducing variable S, which characterizes the potential of the agrobiological system to provide limited yield during the vegetation period, the modified Monod equation system takes the following form:

$$\begin{cases} \frac{d}{dt}X(t) = p_1 \frac{X(t)S(t)}{p_2+S(t)}, \\ \frac{d}{dt}S(t) = -p_3 \frac{X(t)S(t)}{p_2+S(t)}, \end{cases} \quad (1)$$

$$X(0) = X_0, \quad S(0) = S_0 \quad (2)$$

In the presented model, parameter p_1 regulates the interaction intensity between the current indicator value and available system resource, parameter p_2 serves as an indicator of balanced index growth. Parameter p_3 reflects the productivity resource depletion rate, while X_0 fixes the initial value of the studied vegetation index, and S_0 determines the scale of potential productivity.

The parametric identification procedure of equation system (1)-(2) is aimed at establishing optimal parametric values that guarantee maximum consistency between theoretical predictions and empirical observations. Considering the gradual nature of transformations in the studied vegetation phenomena, where jump-like stochastic fluctuations are atypical, the application of a quadratic criterion for quantitative assessment of empirical data approximation quality is rational.

Implementation of this methodological approach enables mathematical formalization of the model parameter calibration algorithm, ensuring adequate reproduction of vegetation index evolution depending on the cumulative GDD indicator. The quadratic criterion (3) guarantees balanced assessment of discrepancies between model and actual values, considering their natural variability without excessive susceptibility to individual anomalies:

$$Q(\vec{p}) = \sum_{j=1}^{N}\left(\tilde{X}\left(t_j,\vec{p}\right) - X_j^e\right)^2 \quad . \quad (3)$$

With determined parameters of the modified Monod model, it becomes possible to calculate system variable values by solving the nonlinear differential equation system (1)-(2). However, in real applications, these parameters usually remain unknown, generating additional methodological challenges.

Special attention is required for parameter p_2, which functions nonlinearly in the system and is characterized by significant variability in the range of possible values. Modification of this parameter fundamentally changes the system solution properties. Meanwhile, the model identification quality functional, constructed on a relatively sparse grid of p_2 values, reveals unimodal characteristics.

A specific feature of parameter p_2 influence is the reduction of its impact on the final result with increasing parameter values. This determines the feasibility of applying a non-uniform grid for effective optimal level search. Particularly, using a grid with increasing step when the parameter increases is effective, for example, according to geometric progression:

$$P_{2,J}^{0} \epsilon \left\{\frac{B}{2} B^J S_0\right\} \quad (4)$$

This methodological approach ensures optimization of computational resources during model parametric identification and guarantees increased accuracy of model result approximation to empirical vegetation index values.

The parameter B value is established empirically to ensure adequate shift of process activity peak when varying parameter p_2 values at grid nodes defined by formula (4). The remaining model parameters are calculated based on the selected parameter p_2 value and approximated difference representation of model differential equations for specific time variable values. The minimum of the unimodal function outlines the search zone for identified model parameters.

In this zone, quality criterion minimization is determined not only by parameter p_2 variations but also by the synergetic influence of all parameters. Therefore, the parameter p_2 value search area is covered by a uniform grid. For each parameter p_2 value on the grid, using difference dependencies, values of other parameters are determined. The obtained values are corrected using a modified gradient algorithm. Among the calibrated parameter values, the one that minimizes the maximum relative error of modeled values compared to observed ones is selected.

When studying equations of system (1)-(2), we note that they include only one undetermined parameter. Having formed an approximated equation representation at one point, it is possible to estimate this parameter value. For this purpose, a point is selected where the vegetation indicator reaches the median value, and its change demonstrates approximately linear dynamics. At this point, derivatives of the indicator and productivity reserve are calculated using specialized dependencies:

$$D_{X,j} = (X^e_{j+1} - X^e_{j-1})/(t^e_{j+1} - t^e_{j-1}), \quad (5)$$

$$D_{S,j} = (S^e_{j+1} - S^e_{j-1})/\left(t^e_{j+1} - t^e_{j-1}\right). \quad (6)$$

The calculated derivative values enable construction of approximated differential equation representation at the moment of reaching the median vegetation indicator value:

$$D_{X,j} \approx p_1 X^e_j S^e_j/\left(p_2 + S^e_j\right), \quad (7)$$

$$D_{S,j} \approx -p_3 X^e_j S^e_j/\left(p_2 + S^e_j\right). \quad (8)$$

Based on these dependencies, model parameter estimates are formed.

The concept of the Monod model identification method is based on systematic enumeration of parameter p_2 values on a uniform grid, for each value of which corresponding initial values of other model parameters are calculated using determined difference dependencies. The obtained initial parameter values are subsequently corrected by the gradient method according to the criterion of minimizing the functional defined by relationship (3).

To construct a uniform grid, a non-uniform grid based on geometric progression (5) is first formed, which serves to determine the base point for subsequent detailed search. The non-uniform grid based on geometric progression is described by the corresponding mathematical representation.

$$W_2(k_{min}, k_{max}) = \left\{\frac{B}{2} B^J S_0\right\} \quad . \quad (11)$$

Construction of the non-uniform grid begins from a point corresponding to half of the initial productivity reserve, since such a parameter p_2 value is an acceptable initial approximation for numerous practically significant processes.

$$W_2(k^0_{min}, k^0_{max}) = \left\{P^0_{4,k_0}\right\}, \quad k^0_{min} = k^0_{max} = k_0 = -1. \quad (12)$$

The presented method enables effective identification of modified Monod model parameters for modeling vegetation index evolution, forming the foundation for integrating remote sensing data into decision support systems in the agricultural sphere.

To reduce discrete model forecast uncertainty, we divide the observation interval into a series $[GDD_0, GDD_{NG}]$of subintervals – forecast windows $\left\{w^{iw} = [w^{iw}_0, w^{iw}_1]\right\}^{Nw}_{iw=0}$.. The distance between the observed part of the current trajectory and an arbitrary trajectory from the statistical set is considered not at individual points, but within the window bounds w^{iw}:

$$d^{iw}(X, x^j) = \sum_{GDD=w^{iw}_0}^{w^{iw}_1} \left|X_{GDD} - x^j_{GDD}\right| \quad (13)$$

As a result, we obtain a set of distances equal to the number J of trajectories from the statistical set, which we order in ascending order:

$$D^{iw}(X) = \tau_{\left\{d^{iw}(X,x^j)\right\}_j} \left(\left\{d^{iw}(X, x^{j_1})\right\}_{j_1}\right) \quad (14)$$

where $\tau -$ is the ordering operator of relational algebra.

From the obtained set B, we select a subset of distances to the nt nearest observed trajectories:

$$B^{iw}(X) = \sigma_{j_1 \le nt}(D^{iw}_{j_1}(X)) \quad (15)$$

and a subset $PI^{iw}(X)$of indices of the nearest trajectories in their initial numbering, where $\sigma -$ is the selection operator of relational algebra.

Next, we find the sum of inverse values of elements from the set: $B^{iw}(X)$

$$E^{iw}_P(X) = \sum_{j_1=1}^{nt} \frac{1}{D^{iw}_{j_1}(X)} \quad (16)$$

and the set of weights for forecast values:

$$W_P^{iw}(X) = \frac{1}{E_P^{iw}(X)}\left\{\frac{1}{D_{j_1}^{iw}(X)}\right\}_{j_1=1}^{nt} \tag{17}$$

Subsequently, we build forecast values of the observed trajectory Y for the next window:

$$P^{iw+1}(X) = \sum_{j_1=1}^{nt} W_P^{iw}(X) x^{PI^{iw}(X)} \tag{18}$$

In this case, similar trajectories within the window are included in forecasting discrete model values for the next window. Forecast values are constructed using the weighted averaging method. After obtaining discrete forecast values, they are interpolated to any point in the forecast interval using the Monod model.

Two-Level Yield Forecasting Model Based on Vegetation Indices

The two-level nature of the model consists in observing values of factors that can significantly influence crop yield, building forecasts of their subsequent values, and at the next stage, building crop yield forecasts based on forecast factor values. Preliminary modelling of yield factors enables building yield factor values for observation points necessary for constructing the yield forecast itself. Vegetation indices NDVI and MTCI are selected as yield factors, which signal the general state of plant development and chlorophyll content in them. A hypothesis is proposed regarding the effectiveness of this type of forecasting model, which requires formalization, software implementation, and practical verification on real data.

As a modelling apparatus for yield factors, we apply the previously described adaptive Monod model. For modelling yield itself based on vegetation indices, we will use alternative approaches in the form of linear regression as well as random forest. Such alternatives allow comparing features of a simple linear approach and an approach that considers non-obvious nonlinear dependencies of yield on its factors.

It should be noted that random forest is built using a set of hundreds or thousands of "decision trees". Each tree learns on a random part of our data, finding its own patterns. The final forecast from random forest is a "collective decision" of all trees. It takes into account not only direct dependency but also all subtle nuances and nonlinear effects that influence yield. An analogy can be used with forecasts not from one meteorologist, but from an entire team, each of whom specializes in a certain field.

Let us assume that we have at our disposal sets of observed values of OV $\left\{X_{NDVI}^e\left(i,t_j\right)\right\}_{i=\overline{1,N_0};j=\overline{1,Tp_i}}$; $\left\{X_{MTCI}^e\left(i,t_j\right)\right\}_{i=\overline{1,N_0};j=\overline{1,Tp_i}}$; trajectories of vegetation index dynamics under implementation of certain crop growing conditions, where, $No -$ is the number of observed crop growing processes, and Tp_i is the number of temporal observation i points. Based on these data, we can build an adaptive two-level yield model. But first, we need to investigate its adequacy. Therefore, we divide the sets of OV observed values, Tr into training, Ts testing, and Cn control subsets:

$$OV = Tr \cup Ts \cup Cn \tag{19}$$

Based on set Tr, we build adaptive models of vegetation indices NDVI and MTCI according to the presented relationships (1)-(18). Next, we build linear regression yield models:

$$Y_L = C_1 X_{NDVI} + C_1 X_{MTCI} \tag{20}$$

or random forest models:

$$Y_{RF} = RF(X_{NDVI},\ X_{MTCI})\ . \tag{21}$$

which we identify based on the set of test observations Ts. Then we control the quality of the built models based on the set of control points that did not participate in model construction and training. Based on the results of model effectiveness analysis on control points, we build recommendations regarding their rational use.

Numerical Experiments

For conducting numerical experiments, we utilize a dataset of trajectories of accumulated vegetation index values observed during rice cultivation, constructed based on materials from work [6]. The study examined index dynamics under normal growing conditions and under drought conditions. Based on these data, a dataset of 21 trajectories of NDVI and MTCI index dynamics was formed, along with a vector of corresponding yields. The first 15 trajectories correspond to normal growing conditions, while the subsequent 6 correspond to heat stress conditions. For illustration purposes, the following figure shows the temporal profiles of observed NDVI vegetation index trajectories, and Figure 2 presents the temporal profiles of observed MTCI vegetation index trajectories.

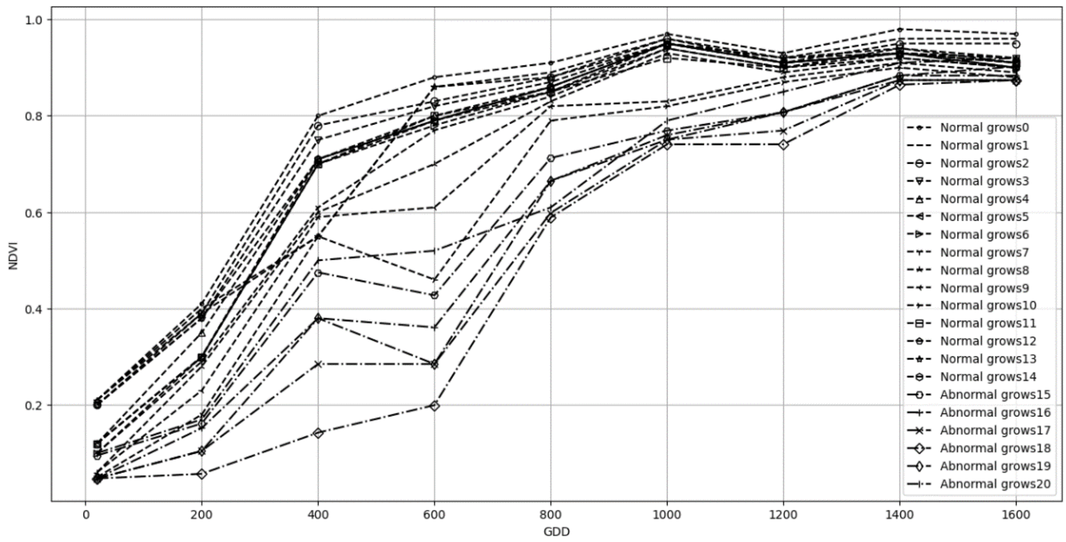

Fig. 1. Temporal profiles of observed NDVI vegetation index trajectories

We can observe sufficiently complex dynamics of the analyzed indices. The construction of the two-level model begins with modelling the dynamics of vegetation indices, specifically the NDVI index.

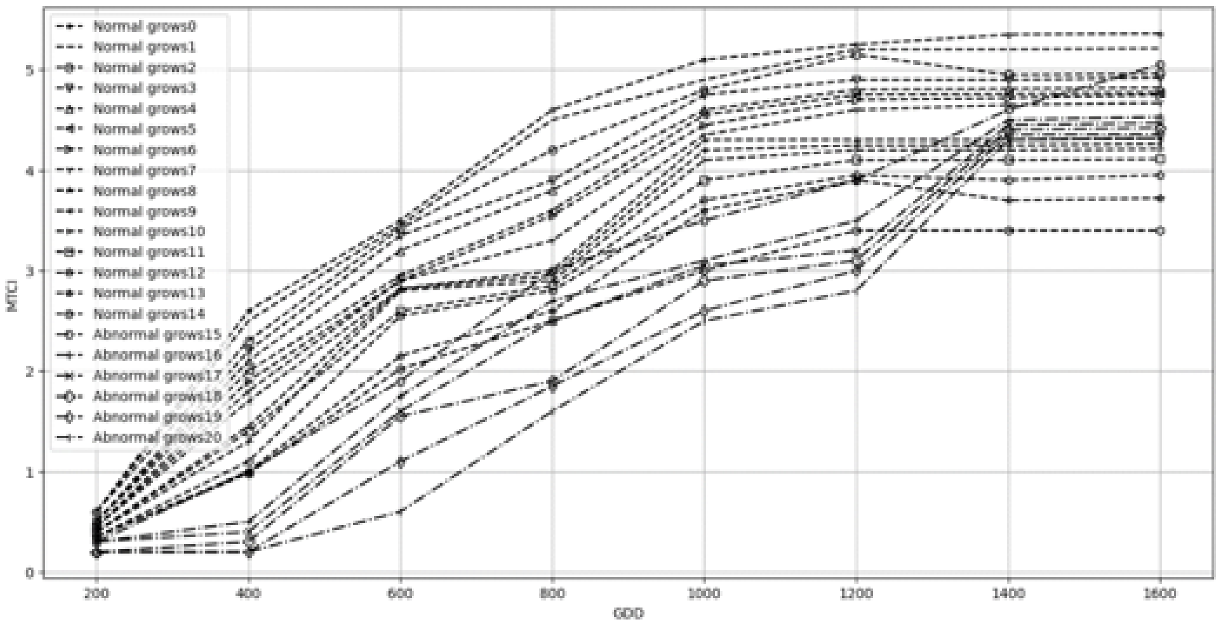

Fig. 2. Temporal profiles of observed MTCI vegetation index trajectories

As the forecasting apparatus, the previously described adaptive Monod model was employed. The modelling results for the zero trajectory (normal growth conditions) and the seventeenth trajectory (heat stress conditions) are presented in Figure 3. A sufficiently good approximation with individual deviations is observed, characterized by a maximum relative error of 7.3% and a mean relative error of 1.3% for the zero trajectory, as well as a maximum relative error of 10.9% and a mean relative error of 1.8% for the trajectory formed under heat stress conditions.

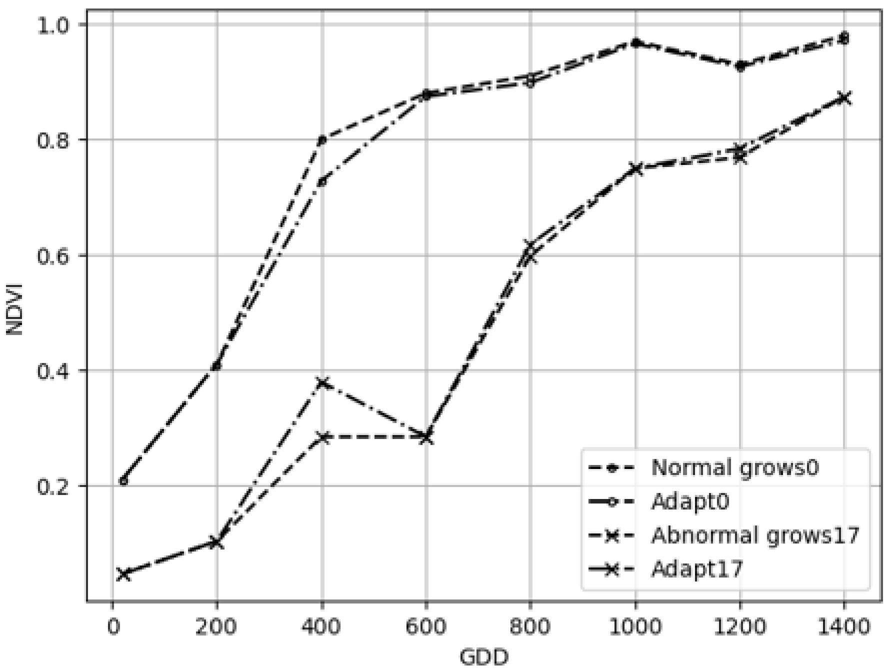

Fig. 3. Adaptive Monod models of NDVI vegetation index dynamics under normal conditions (zero trajectory) and heat stress conditions (seventeenth trajectory)

In the next stage, we investigate the effectiveness of the MTCI vegetation index dynamics model. The modelling results for the zero trajectory (normal growth conditions) and the seventeenth trajectory (heat stress conditions) are presented in Figure 4. A sufficiently good approximation with minor deviations is observed, characterized by a maximum relative error of 0.9% and a mean relative error of 0.4% for the zero trajectory, as well as a maximum relative error of 7.5% and a mean relative error of 2.6% for the trajectory formed under heat stress conditions. As can be seen, the adaptive model provides quite accurate predictions of vegetation index dynamics.

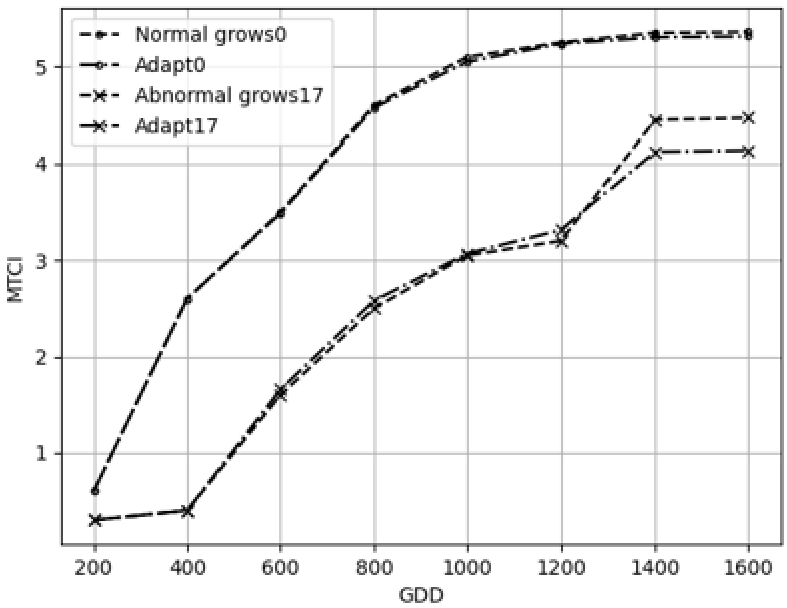

Fig. 4. Adaptive Monod models of MTCI vegetation index dynamics under normal conditions (zero trajectory) and heat stress conditions (seventeenth trajectory)

We proceed to modelling yield based on vegetation indices using linear regression and random forest approaches. Of the 21 observations, 14 were used for training, 3 observations for testing, and 4 for validation. Figure 5 presents a 3D plot of predicted yield based on the linear regression model.

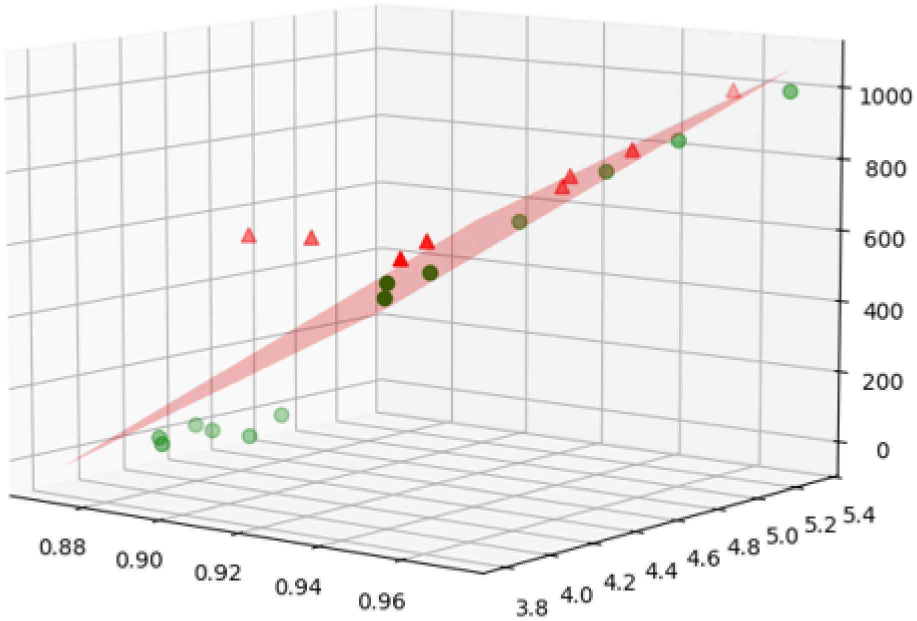

Fig. 5. 3D plot of predicted yield based on the linear regression model

In constructing this model, the LinearRegression class from the Python Sklearn library was used. For better visualization, observations lying above the model hyperplane are marked with triangles, while observations lying below the hyperplane are marked with circles. The maximum relative error of the model on the validation set was 17.7%, and the mean relative error was 8.3%.

The random forest model allows for much more accurate modelling of nonlinear data relationships, the results of which are presented in Figure 6.

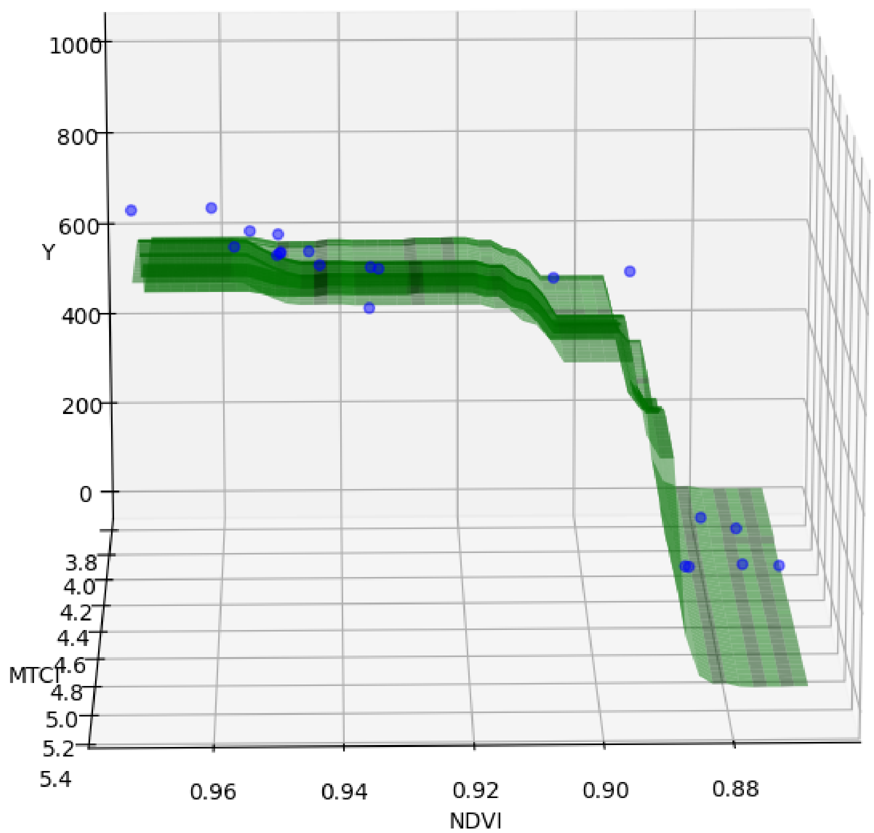

Fig. 6. 3D plot of predicted yield based on the random forest model

In constructing this model, the RandomForestRegressor class from the Python Sklearn library was used. The maximum relative error of the model on the validation set was 14.1%, and the mean relative error was 5.3%.

Conclusions

This work analyzes an innovative approach to modelling crop yields based on remotely observed vegetation index values using a secure information architecture. A two-level model has been developed and experimentally validated, where dynamic characteristics of vegetation indices are modeled at the first level, and yield indicators are predicted at the second level through linear regression and random forest models to account for nonlinear relationships. Vegetation index modelling is implemented using an adaptive modified Monod model considering the principle of irreversibility in vegetation characteristic accumulation.

The optimal structure of the Monod equation system has been established, adequately reflecting the nature of empirical observations of vegetation process dynamics. An efficient method for parametric identification of the constructed system has been developed based on the criterion of minimizing mean squared error, with a key feature being an algorithm for generating initial model parameter values considering its nonlinear nature. Further refinement of initial parametric estimates was performed using the Levenberg-Marquardt gradient method, ensuring high approximation accuracy.

The practical effectiveness of the proposed models is confirmed by numerical experiment results on real rice vegetation index data. For the linear regression model, a maximum relative error of 17.7% and a mean relative error of 8.3% were recorded on the validation set. The random forest model demonstrated higher accuracy with a maximum relative error of 14.1% and a mean relative error of 5.3%, confirming the feasibility of accounting for nonlinear dependencies between vegetation indices and yield.

An adaptive prediction algorithm based on observation windows has been developed, enabling effective utilization of an ensemble of previously observed trajectories to improve forecast accuracy under conditions of limited initial observations. Application of the weighted averaging method considering trajectory proximity ensures model robustness to variations in vegetation processes.

The accuracy of obtained results and their practical applicability demonstrate the promise of the proposed approach for solving current problems in precision agriculture, food security monitoring, and making informed management decisions in the agricultural sector. Future research should focus on expanding the set of vegetation indices, integrating meteorological factors, and adapting the model for different types of agricultural crops and climate zones.

Author Contributions according to CRediT

Author Contributions: Conceptualization, R.P., M.M.; methodology, R.P., M.M.; software, M.M.; validation, R.P.; formal analysis, R.P. and M.M. ; investigation, R.P. and M.M.; resources, R.P. and M.M.; data curation, M.M. ; writing—original draft preparation, ; writing—review and editing, R.P. and M.M.; visualization, M.M.; supervision, R.P.; project administration, R.P. All authors have read and agreed to the published version of the manuscript.

Declaration on the use of generative artificial intelligence tools

In preparing this work, the authors used Claude.ai for: grammar and spelling checks. After using this tool, the authors reviewed and edited the content and takes full responsibility for the content of this publication.

References

1. Cheng Y.B., Zhang Q., Lyapustin A.I., Wang Y., Middleton E.M. Impacts of light use efficiency and fPAR parameterization on gross primary production modeling. *Agricultural and Forest Meteorology*. 2014. Vol. 189. P. 187-197. DOI: https://doi.org/10.1016/j.agrformet.2014.01.006
2. Yang Z., Shao Y., Li K., Liu Q., Liu L., Brisco B. An improved scheme for rice phenology estimation based on time-series multispectral HJ-1A/B and polarimetric RADARSAT-2 data. *Remote Sensing of Environment*. 2017. Vol. 195. P. 184-201. DOI: https://doi.org/10.1016/j.rse.2017.04.016
3. Ko J., Jeong S., Yeom J., Kim H., Ban J.-O., Kim H.-Y. Simulation and mapping of rice growth and yield based on remote sensing. *Journal of Applied Remote Sensing*. 2015. Vol. 9, No. 1. Art. 096067. DOI: https://doi.org/10.1117/1.JRS.9.096067
4. Yeom J.M., Jeong S., Jeong G., Ng C.T., Deo R.C., Ko J. Monitoring paddy productivity in North Korea employing geostationary satellite images integrated with GRAMI-rice model. *Scientific Reports*. 2018. Vol. 8, No. 1. Art. 16121. DOI: https://doi.org/10.1038/s41598-018-34550-0
5. Mosleh M., Hassan Q., Chowdhury E. Application of remote sensors in mapping rice area and forecasting its production: A review. *Sensors*. 2015. Vol. 15, No. 1. P. 769-791. DOI: https://doi.org/10.3390/s150100769
6. Ryu J.-H., Jeong H., Cho J. Performances of Vegetation Indices on Paddy Rice at Elevated Air Temperature, Heat Stress, and Herbicide Damage. *Remote Sensing*. 2020. Vol. 12, No. 16. Art. 2654. DOI: https://doi.org/10.3390/rs12162654
7. Liu Y., Lin Y.-M., Yang S.-F. A Thermodynamic Interpretation of the Monod Equation. *Current Microbiology*. 2003. Vol. 46, No. 3. P. 233-234. DOI: https://doi.org/10.1007/s00284-002-3898-2
8. Dette H., Melas V., Strigul N. A practical guide for optimal designs of experiments in the Monod model. *Technische Universität Dortmund*. 2006. 25 p. DOI: https://doi.org/10.17877/DE290R-14448
9. Pasichnyk R., Pigovsky Y., Gunderych O. Accelerating the Parametric Identification of the Monod System Using Explicit and Implicit Schemas. *Proceedings of the International Conference on Modern Problems of Radio Engineering, Telecommunications and Computer Science (TCSET'2012)*. Lviv-Slavske, Ukraine, February 21-24, 2012. P. 285-286.

10. See J.J., Jamaian S.S., Salleh R.M., Nor M.E., Aman F. Parameter estimation of Monod model by the Least-Squares method for microalgae Botryococcus Braunii sp. *Journal of Physics: Conference Series*. 2018. Vol. 995, No. 1. Art. 012026. DOI: https://doi.org/10.1088/1742-6596/995/1/012026

11. Molin H. Optimal steady-state design of bioreactors in series with Monod growth kinetics. *Master thesis*. Uppsala: Uppsala University, 2017. 47 p.

12. Pasichnyk R., Babala L., Machulyak M., Yakymenko Y. Modeling Vegetation Index Dynamics in GIS Based on Remote Observations. The Second International Conference of Young Scientists on Artificial Intelligence for Sustainable Development (YAISD), May 8-9, 2025, Ternopil, Ukraine

13. Pasichnyk R.M., Babala L.V., Machuliak M.V. A method for improving the quality of image annotation in semantic monitoring GIS of business processes. *Informatics & Mathematical Methods in Simulation*. 2024. Vol. 14, No. 3. P. 195-203.

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