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SEAMLESS TILING OF QUASI-PERIODIC TEXTURES VIA AN OPTIMAL CYCLIC SHIFT ON A DISCRETE TORUS

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In practical computer vision and computer graphics pipelines, it is often necessary to repeatedly replicate a single texture sample to construct a large canvas, background, or regular covering. When the mosaic is not strictly periodic, visible seams appear at the boundaries during repetition, disrupting the perceptual continuity of the texture and often manifesting as a regular grid of artifacts. Such seams not only degrade visual quality but can also alter local gradients and spectral components, which is critical for subsequent processing stages. Common seamless stitching methods increase computational complexity, introduce additional hyperparameters, and modify the local image statistics, which is undesirable in reproducible pipelines and in tasks where the invariance of pixel values is essential. The goal of this work is to propose a simple, reproducible, and computationally efficient method for seam reduction in quasiperiodic textures by selecting an optimal cyclic shift of the pattern that minimizes the energy of mismatch between opposite boundaries. The tile is modeled as a function on the discrete torus $\mathbb{Z}_M \times \mathbb{Z}_N$. A cyclic shift group $G = \mathbb{Z}_M \times \mathbb{Z}_N$ is introduced, acting as a permutation of pixels. For each shift $\tau_{a,b}$, the boundary seam energy $E(\tau_{a,b}I)$ is computed in a band of width w for opposite boundary pairs, and the minimizing shift is selected. When needed, the evaluation is accelerated via cyclic correlations and FFT. Experiments on synthetic and real textures show that the optimal cyclic shift significantly reduces seam energy and the visual prominence of boundaries during tiling without modifying pixel values. For strictly periodic tiles, the method does not degrade the result. The proposed approach is a lightweight baseline tool for seamless tiling: it does not perform stitching but selects the best cut of the torus. The method is easy to integrate into production pipelines and can be used as a preprocessing step before further processing.

Keywords: seamless tiling, quasi-periodic textures, optimal cyclic shift, discrete torus, minimization of boundary artifacts, periodic boundary conditions, FFT-based optimization

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Introduction

In modern computer vision systems and digital content production, there is a recurring need to work not only with ultra-high resolutions but also with repeated image fragments. This need arises in practical tasks of synthesizing large textures and materials for rendering, in generating mosaics from texture samples, in building training datasets and performing data augmentation, as well as in reproducible tests for comparing filters and spectral procedures. In these scenarios, the typical operation consists in repeatedly copying one or several basic fragments, referred to as tiles, along the axes of a grid in order to obtain a large visual canvas without noticeable seams.

At first glance, it may seem sufficient to select a symmetric or visually compatible fragment so that seams disappear. In practice, however, symmetry does not guarantee periodic consistency of opposite boundaries under repetition. Even when boundary pixels coincide, a seam may appear due to mismatches of gradients or local statistics in near-boundary regions, as well as because subsequent processing steps amplify the smallest discontinuities at the junction. As a result, visible seams arise in mosaics and texture canvases. These seams are not only an aesthetic defect but also a source of systematic distortions: they introduce spurious high-frequency components, alter phase and spectral characteristics, and generate false gradients and contours. This negatively affects subsequent segmentation, detection, and spectral processing algorithms, particularly those relying on FFT and assuming boundary consistency.

Therefore, there is a clear need for a reproducible and computationally simple approach that eliminates or significantly attenuates seams specifically in the tile repetition scenario, without modifying pixel values and without resorting to heavy blending schemes. In this work, this problem is addressed through a toroidal interpretation of the tile and the use of a cyclic shift group as a formal mechanism for selecting the best cut for quasiperiodic textures.

Figure 1 illustrates how even minor discrepancies in intensity and geometric structures at the boundaries of the initial tile, when periodically repeated, transform into a regular grid of visual artifacts that disrupt the perceptual integrity of the texture.

Fig. 1 Examples of artifacts arising from periodic repetition of a tile

The object of the study is the process of forming an integral texture canvas or mosaic through repeated replication of one or several base fragments in systems of visual information processing and synthesis.

The subject of the study is mathematical models and algorithmic procedures for reducing boundary artifacts during tile repetition, based on the action of a cyclic shift group on a discrete torus and on optimizing the choice of tile cut without modifying pixel values.

The aim of the work is to develop and justify a reproducible and computationally efficient method for seamless tiling of quasiperiodic textures by finding an optimal cyclic shift of the tile that minimizes the energy of discontinuity between opposite boundaries within a band of prescribed width.

To achieve this aim, the following tasks are addressed:

1. To formulate a model of a tile as a function on the discrete torus $\mathbb{Z}_M \times \mathbb{Z}_N$ and to define the action of the cyclic shift group as permutations of indices.
2. To define a reproducible seam energy functional in near-boundary regions that reflects both intensity discontinuities and derivative or gradient discontinuities amplified by subsequent processing.
3. To develop an algorithm for finding the optimal shift on the group $\mathbb{Z}_M \times \mathbb{Z}_N$ and to justify the possibility of accelerating it using cyclic correlations and FFT.
4. To conduct experimental validation on synthetic and natural textures and to compare the proposed approach with baseline methods by seam energy metrics and invariance of local statistics.

State-of-art

In seamless tiling and image stitching tasks for textures, mosaics, and panoramas, three families of approaches are used most often: (i) overlap and blending in the seam region, (ii) mirror and reflective boundary conditions, (iii) gradient-domain variational methods based on the Poisson equation. All three families indeed reduce seam visibility, but they do so by different mechanisms and therefore have fundamental limitations that are critical for reproducible and analytical computer-vision pipelines.

Overlap-and-blend approaches reduce the seam to a controlled transition zone, where pixel values are a weighted combination of neighboring fragments in the spatial domain or in a pyramidal frequency-domain representation. A classical example is multiresolution blending based on Laplacian pyramids for mosaics and panoramas [1,2,3]. In texture synthesis, patch-based schemes are widely used, where overlap minimizes local mismatch and can be combined with an optimal seam cut in the overlap region [4,5]. However, a common consequence of these methods is that they inevitably change photometric values in boundary bands and in the overlap region, and thus change local statistics, including means, variances, histograms, and local correlations. This can be unacceptable in scenarios where the measurement fidelity of intensities is part of the task, including quantitative microstructural characterization, medical imaging, and validation of filters and spectral procedures, as well as in pipelines whose results are sensitive to small systematic changes in boundary regions.

Mirror tiling and reflection are often used as simple boundary conditions for local filtering and convolution, since they provide a soft continuation of intensity and reduce the boundary jump compared to zero padding [6,7,8]. However, in tiling as synthesis, where one fragment is repeatedly copied over the plane, the mirror rule imposes a global axial symmetry: it changes the natural directionality of the texture, produces regular symmetry axes, and creates artificial repeating patterns. Such symmetries can be undesirable not only visually but also algorithmically, because many descriptors and analysis models distinguish anisotropy and oriented structures.

Gradient-domain Poisson schemes formalize seamlessness as gradient consistency and recover intensities from the solution of the Poisson equation with corresponding boundary conditions [9]. This yields a strong seam-suppression effect and is a standard tool for tasks such as seamless cloning and image editing. At the same time, these methods solve an additional optimization or elliptic partial differential equation problem, have a significant computational cost, and introduce implicit assumptions about smoothness and the choice of boundary conditions. In real-time industrial pipelines this complicates integration, and in the context of reproducibility it shifts the seam model from an explicit rule to hidden parameters and numerical aspects of the solver.

A separate body of work emphasizes that many frequency-domain procedures, including the discrete Fourier transform, phase correlation, and circular convolution, inherently rely on a periodic signal model: the discrete Fourier representation naturally diagonalizes cyclic shifts and circular convolution, but these properties are correct precisely for periodic extension [10, 11, 12]. This leads to a methodological conclusion: when a pipeline includes cyclic and frequency-domain operators, the choice of boundary model is not cosmetic but determines result consistency, including phase and spectral characteristics. For this reason, an approach that does not smooth the seam at the cost of changing data, but instead explicitly controls cyclic symmetry through shifts and permutations on the discrete torus, is promising for tasks with quasiperiodic textures and reproducible transformations.

At the same time, the class of lightweight approaches that do not perform smoothing and do not modify pixel values, but affect only the geometric representation of the tile, remains insufficiently explored. In particular, if a tile is interpreted as a discrete function on the torus $\mathbb{Z}_M \times \mathbb{Z}_N$, then cyclic shifts acquire a clear mathematical meaning as actions of a permutation group on indices. In this formulation, seam elimination can be treated not as post hoc repair of a junction, but as the search for an optimal cut of the torus, that is, a choice of rectangular representation for which opposite boundaries are maximally consistent according to a prescribed metric. This allows reducing the regular grid of artifacts in a mosaic while preserving pixel values and local statistics, and at the same time ensures consistency with procedures where cyclic shifts and periodic models are natural, such as FFT-based constructions.

Problem Statement

Let a discrete image (tile) be represented as a function $I(i,j)$ defined on the integer grid $\Omega = \mathbb{Z}_M \times \mathbb{Z}_N$, where $M, N \in \mathbb{N}$ are the tile height and width, respectively. The values $I(i,j) \in \mathbb{R}^c$ correspond to pixel intensities in a c-channel color space.

Input data:

1. The input pixel array I of size $M \times N \times c$.
2. The boundary-band width parameter $w \in \mathbb{N}$ such that $2w < \min(M, N)$.
3. The gradient-term weight $\lambda \geq 0$.

Definition of the shift operator: The domain Ω is interpreted as a discrete torus $\mathbb{T}^2_{M,N}$. Define the group of cyclic shifts $G = \{\tau_{a,b} \mid a \in \mathbb{Z}_M,\ b \in \mathbb{Z}_N\}$, where the action of $\tau_{a,b}$ on an image I is given by

$$(\tau_{a,b}I)[i,j] = I[(i-a) \pmod M,\ (j-b) \pmod N].$$

Quality criterion (objective function): To quantify signal continuity across tile boundaries under periodic repetition, we introduce the seam-energy functional $E_{\text{total}}(\tau_{a,b}I)$ consisting of two components:

1. Intensity-discontinuity energy $E_{\text{int}}(J) = E_{\text{LR}}(J) + E_{\text{TB}}(J)$, where

$$E_{\text{LR}}(J) = \sum_{i=0}^{M-1}\sum_{k=0}^{w-1} \| J[i,k] - J[i, N-w+k] \|^2,$$

$$E_{\text{TB}}(J) = \sum_{j=0}^{N-1}\sum_{k=0}^{w-1} \| J[k,j] - J[M-w+k, j] \|^2.$$

2. Gradient-discontinuity energy $E_{\text{grad}}(J)$, computed analogously to E_{int} for the discrete difference fields $\nabla_x J$ and $\nabla_y J$.

The full functional is

$$E_{\text{total}}(J) = E_{\text{int}}(J) + \lambda\, E_{\text{grad}}(J).$$

Desired output: Find an optimal shift vector $(a^*, b^*) \in G$ minimizing the total seam energy:

$$(a^*, b^*) = \arg\min_{(a,b)\in\mathbb{Z}_M\times\mathbb{Z}_N} E_{\text{total}}(\tau_{a,b}I).$$

The output is the transformed tile $I^* = \tau_{a^*,b^*}I$, which yields minimal visual and spectral boundary salience under tiling.

Constraints:

- The operator $\tau_{a,b}$ is a permutation preserving the multiset of values $\{I[i,j]\}$; hence $\| I^* \|_p = \| I \|_p$ for any L_p norm.
- The problem is solved within the discrete torus topology, without pixel-value interpolation.

Theoretical Foundations and Mathematical Framework

Tiling is the process of forming a continuous visual canvas by repeatedly copying a base fragment (tile) I of size $M \times N$ along the axes of a Cartesian grid. Mathematically, toggle the mosaic $\mathcal{K}$ can be represented as the mapping

$$\mathcal{K}[i,j] = I[i \ (\mathrm{mod}\ M),\ j \ (\mathrm{mod}\ N)],$$

where (i,j) are coordinates on the infinite discrete plane $\mathbb{Z}^2$.

In practical digital image processing tasks, the source tile rarely possesses the property of strict periodicity. When pixel values on opposite boundaries differ substantially, that is,

$$I[i,0] \neq I[i,N-1] \quad \text{or} \quad I[0,j] \neq I[M-1,j],$$

boundary artifacts arise at the interfaces of adjacent copies, commonly referred to as seams. This problem has two key aspects:

1. Geometric discontinuity (C^0): a sharp jump in intensity or chromaticity that is perceived by the visual system as a linear anomaly.
2. Structural discontinuity (C^1): a mismatch of gradients and contour directions. Even when the mean brightness of the boundaries is identical, a break of textural lines generates spurious high-frequency components in the image spectrum and leads to spectral leakage.

To formalize operations on a tile, this work adopts a transition from representing the image on a bounded planar domain to interpreting it on the discrete torus $\mathbb{T}^2_{M,N}$. The discrete torus is defined as the Cartesian product of two cyclic groups:

$$\mathbb{T}^2_{M,N} = \mathbb{Z}_M \times \mathbb{Z}_N,$$

where $\mathbb{Z}_n = \{0,1,\dots,n-1\}$ is the group of integers modulo n.

Figure 2 schematically illustrates the topological identification of opposite edges of a rectangular domain, resulting in a closed torus surface without physical boundaries.

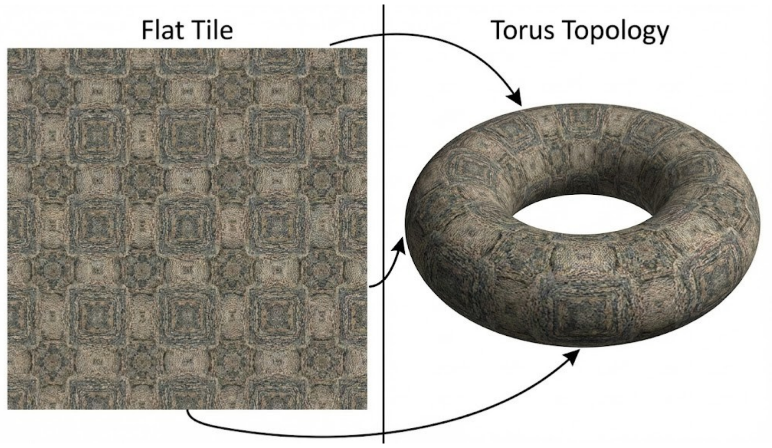

Fig. 2 Visualization of the discrete torus topology: arrows indicate the directions of identifying opposite edges of a two-dimensional tile, which transforms it into a closed surface.

Such a topological model implies that the notion of an image boundary exists only in the visual representation of the rectangular unfolding, whereas algebraically each pixel has an identical neighborhood configuration. For example, for a pixel in the last column $(i, N-1)$, its right neighbor is defined to be the pixel in the first column $(i,0)$. This removes the ambiguity of defining local operators near domain boundaries.

The main transformations in the proposed model are cyclic shifts. Unlike linear shifts on the plane, which require padding of empty regions, a cyclic shift on the torus is an automorphism that preserves the set of signal values.

For an arbitrary shift vector $(a,b) \in \mathbb{T}^2_{M,N}$, the operator $\tau_{a,b}$ maps each pixel to a new position according to the rule

$$(i',j') = (i + a \ (\mathrm{mod}\ M),\ j + b \ (\mathrm{mod}\ N)).$$

An important property is that circular convolution, which underlies the Fast Fourier Transform, is naturally consistent with this structure. Any linear filtering on the torus is equivalent to processing a signal that has been periodically extended. Therefore, choosing the toroidal model makes the system invariant to cyclic shifts, which allows one to treat the seamless stitching task as a problem of finding an optimal cut of the torus.

The basic mathematical apparatus used in this section, including discrete convolutions and their boundary models, linear shift operators, circular convolutions on $\mathbb{Z}_M \times \mathbb{Z}_N$, and the discrete Fourier transform together with its connection to cyclic shifts and correlation, is standard and is described in detail in classical textbooks on digital signal processing and digital image processing. In this work, we provide only those definitions and formulations that are necessary for the subsequent toroidal formulation and optimization of the tile cut. For reference, the reader is directed to [6,7,10,11,12].

Proposed technique

In this work, we test the following working hypothesis: let the tile $I \in \mathbb{R}^{M\times N}$ be quasiperiodic in the sense that there exists a representation of the tile as an unfolding of the discrete torus $\mathbb{Z}_M \times \mathbb{Z}_N$ for which the opposite edges are statistically closer than in the original cut. Then there exists a cyclic shift $\tau_{a,b}$ that minimizes a measure of boundary mismatch between the left–right and top–bottom edges and, as a consequence, reduces visible seams under mosaic repetition of the tile.

The proposed approach does not perform stitching and does not change pixel values; it only selects a different torus cut by permuting indices through a cyclic shift. This distinguishes it from blending-based and Poisson-based stitching approaches, which modify photometry and local statistics. Stitching and overlap methods are previously known and are not detailed in this section; they are used only as baseline comparison procedures with references provided in the literature review.

The object of study is a single tile (a texture fragment) I and the result of its mosaic repetition. The tile is a discrete multichannel field

$$I: \{0, \dots, M-1\} \times \{0, \dots, N-1\} \to \mathbb{R}^c,$$

where $c \in \{1,3\}$ for the grayscale or color case.

To ensure validity and generalizability of the conclusions, we use two complementary classes of data. The first class consists of synthetic controlled textures, both strictly periodic and quasiperiodic, which allow explicit control of the degree of nonperiodicity of the signal and enable verification of the method on limiting cases; in particular, for strictly periodic tiles one expects zero or practically zero seam energy regardless of the cut. The second class consists of real texture patches that reflect typical practical scenarios in which the tile is not strictly periodic and visible seams arise precisely due to an unfavorable choice of cut when repeating the fragment.

To ensure reproducibility of the results across different data sources, the experimental protocol fixes all key processing steps. For color images with $c = 3$, seam energies are computed either per channel with subsequent averaging or in a luminance space, for example, using the Y component in the $YCbCr$ space; the specific choice is fixed and not changed within a series of experiments. Pixel values are converted to real numbers and normalized to the interval $[0,1]$. For synthetic textures, all generation parameters, including spatial frequencies, amplitudes, and noise level, are specified deterministically using a fixed pseudorandom number generator seed, which ensures full repeatability of the experiments.

The key structure is the discrete torus as an index set with modular arithmetic:

$$\mathbb{Z}_M = \{0, \dots, M-1\}, \qquad \mathbb{Z}_N = \{0, \dots, N-1\}.$$

Index periodicity is defined by the operation

$$\operatorname{mod}(k, M) \in \{0, \dots, M-1\}.$$

For parameters $(a, b) \in \mathbb{Z}_M \times \mathbb{Z}_N$, define the operator

$$(\tau_{a,b} I)[i,j] = I[(i-a) \bmod M,\ (j-b) \bmod N].$$

The set $\{\tau_{a,b}\}$ forms a finite abelian group

$$G_{M,N} \cong \mathbb{Z}_M \times \mathbb{Z}_N,$$

which acts on the set of tile pixels as permutations of coordinates. The property of not changing pixel values is fundamental in this approach because it guarantees the absence of photometric distortions introduced by stitching methods.

Consider the tile as an unfolding of the torus to a rectangle. The cut determines which torus points appear on the left–right and top–bottom boundaries of the rectangle. The shift $\tau_{a,b}$ is equivalent to moving the cut: it changes the sets of boundary pixels but does not change the toroidal structure itself. Therefore, minimizing the seam energy over $G_{M,N}$ is a formal formulation of selecting the least conflicting cut.

To evaluate the seam, we consider pairs of opposite edges not pointwise but within a band of width w, which reduces sensitivity to individual pixels and better reflects the visual nature of seams. The value w is a method parameter; in experiments it is fixed in the range $w \in [4,16]$ depending on the tile size, typically $w = \lfloor \min(M, N)/32 \rfloor$ with clipping to $[4,16]$.

For a single channel, or for a norm across channels, define the edge-mismatch energy.

Left–right:

$$E_{\mathrm{LR}}(I; w) = \sum_{i=0}^{M-1} \sum_{j=0}^{w-1} \| I[i,j] - I[i, N-w+j] \|_2^2.$$

Top–bottom:

$$E_{\mathrm{TB}}(I; w) = \sum_{j=0}^{N-1} \sum_{i=0}^{w-1} \| I[i,j] - I[M-w+i, j] \|_2^2.$$

Total seam energy:

$$E(I; w) = E_{\mathrm{LR}}(I; w) + E_{\mathrm{TB}}(I; w).$$

The quantity $E(I; w)$ equals zero for a strictly periodic tile in the sense of matching corresponding boundary bands and increases with the degree of nonperiodicity. It is also invariant to adding a constant when the comparison is performed as a difference, and it is sensitive to local contrast discontinuities, which matches the goal of suppressing visually salient seams.

Since the visual salience of seams is often determined not only by intensity jumps but also by discontinuities of derivatives, we introduce a gradient measure. We use forward differences:

$$(\nabla_x I)[i,j] = I[i,j+1] - I[i,j], \qquad (\nabla_y I)[i,j] = I[i+1,j] - I[i,j],$$

where indices are computed inside the tile; values on the right and bottom boundaries can be omitted or defined on the torus, and one option is fixed in the protocol.

Then

$$E_\nabla(I;w) = E(\nabla_x I;w) + E(\nabla_y I;w).$$

This metric separately controls the smoothness of the transition across the seam and is important for subsequent operators that use gradients.

The main optimization function is defined as

$$E_\lambda(I;w) = E(I;w) + \lambda\, E_\nabla(I;w), \qquad \lambda \geq 0.$$

The parameter λ sets the trade-off between matching intensities and matching derivatives. In all experiments, λ is fixed, typically in the range $\lambda \in [0.1,1]$; the choice is justified empirically on a validation subset of synthetic textures to avoid tuning to the test examples.

The proposed method consists in finding a shift that minimizes the seam energy:

$$(a^*, b^*) = \arg\min_{(a,b)\in\mathbb{Z}_M\times\mathbb{Z}_N} E_\lambda(\tau_{a,b} I;w), \qquad I^* = \tau_{a^*,b^*} I.$$

The resulting tile I^* is used for repetition on the plane without additional stitching. If the tile is strictly periodic, then $E(I;w) = 0$ and the problem has trivial minima; in this case, the method does not degrade the result.

For each tile I, the following sequence of steps is performed:

1. Parameter selection: choose w and λ according to a predefined rule.
2. Quality map computation: for all $(a,b) \in \mathbb{Z}_M \times \mathbb{Z}_N$, compute
3. $S[a,b] = E_\lambda(\tau_{a,b} I;w)$.
4. Optimum selection: find $(a^*, b^*) = \mathrm{argmin} S[a,b]$ and, in case of ties, choose the lexicographically smallest pair for determinism.
5. Tile construction: form $I^* = \tau_{a^*,b^*} I$.
6. Evaluation: build a $K \times K$ mosaic, typically with $K = 4$, from I and from I^* and evaluate the metrics E and E_∇ on mosaic boundaries, and record visual examples.

This protocol contains no stochastic steps except for synthetic data generation with a fixed seed, which ensures reproducibility.

A naive implementation without acceleration computes $S[a,b]$ by direct summation within bands of width w. The complexity is estimated as

$$O(MN \cdot w(M+N)),$$

because for each of the MN shifts one must compare $O(wM + wN)$ pixels from two bands. This variant is chosen as the baseline because it is transparent and contains no hidden assumptions. All results can be reproduced without FFT libraries.

To preserve the applied nature of the work, FFT acceleration is treated as an engineering improvement that does not change the formulation. It relies on the fact that the quadratic energy of the difference of two bands,

$$\sum(A-B)^2,$$

decomposes into a sum of squares and a correlation term:

$$\sum(A-B)^2 = \sum A^2 + \sum B^2 - 2\sum AB.$$

The first two terms do not depend on the mutual shift, whereas the third term is a cyclic correlation of bands that can be computed via FFT. The same is done for horizontal bands. As a result, the map $S[a,b]$ can be computed in

$$O(MN\log(MN))$$

using standard FFT procedures. The experimental protocol records whether acceleration is used; the resulting (a^*, b^*) must coincide for the naive and FFT variants, which serves as an additional correctness test.

Quality assessment is performed in two ways: by direct evaluation of tile-edge energies for I and I^*, and by evaluating seams on a mosaic that models practical repetition. The main indicators are

$$E(I;w), \quad E_\nabla(I;w), \quad E(I^*;w), \quad E_\nabla(I^*;w),$$

as well as the relative improvement

$$\Delta_E = \frac{E(I^*;w)}{E(I;w)+\varepsilon}, \qquad \Delta_{E_\nabla} = \frac{E_\nabla(I^*;w)}{E_\nabla(I;w)+\varepsilon},$$

where $\varepsilon > 0$ is a small stabilizer, for example 10^{-12}, that prevents division by zero on nearly periodic examples.

For mosaic-level evaluation, we build a $K \times K$ mosaic by repeating the tile:

$$\mathcal{M}_K(I) = \quad \{I \;\|\; \dots \;\|\; I\},$$

where || denotes concatenation. Similarly, $\mathcal{M}_K(I^*)$ is constructed. We then compute seam energies not on the outer boundary of the mosaic but on the internal boundaries between copies. This removes boundary effects due to mosaic cropping and reflects repetition itself.

Correctness and reliability of the experiments are ensured by a set of control checks recorded in the protocol. First, invariance to pixel permutation is verified: since the optimal shift is only a reindexing, the value histogram or the total energy $\sum I^2$ must coincide for the original tile I and the shifted tile I^*. Second, a strict periodicity control case is used: for synthetic periodic textures one expects $E(I;w) = 0$ up to machine precision and no degradation after any cyclic shift, which confirms correct implementation of the metrics and the operator $\tau_{a,b}$. Third, when FFT acceleration is applied, agreement with the naive exhaustive search is verified on a subset of test tiles, meaning that the found parameters (a^*, b^*) are identical in both implementations, which excludes implementation errors. Fourth, stability under intensity scaling is controlled: under $I \mapsto \alpha I$, the energies E and E_∇ scale as α^2, so normalized relative indicators Δ_E and Δ_{E_∇} are used to compare different tiles, and these indicators are invariant to such scaling.

Experiments

The experimental evaluation is aimed at a reproducible assessment of how much the optimal cyclic shift of a tile reduces the visibility of seams under mosaic repetition and how this reduction agrees with the quantitative metrics $E(\cdot;w)$ and $E_\nabla(\cdot;w)$ defined in the Materials and methods section. All experiments are performed in a digital environment on a standard workstation, a personal computer running Linux or Windows with a CPU and at least 16 GB of RAM; no specialized hardware is used. The algorithms are implemented in Python version 3.10 or higher using common numerical and image-processing libraries, NumPy and OpenCV or an equivalent library, and for FFT acceleration a standard FFT implementation is used, NumPy FFT or FFTW via an appropriate wrapper. To ensure repeatability in the case of synthetic generation, a fixed pseudorandom number generator seed is set; all parameters, including the tile sizes, the band width w, the weight λ, and the mosaic size K, are fixed and are not tuned to individual examples.

The experimental data consist of two sources. The first source is synthetic controlled textures that allow one to vary the degree of periodicity in a controlled manner. Strictly periodic samples are constructed as superpositions of two-dimensional sinusoids and or lattice patterns with integer spatial frequencies consistent with the tile size $M \times N$ to ensure exact periodicity at the boundaries. Quasiperiodic samples are obtained by modifying periodic fields through the addition of one or more local defects, for example Gaussian blobs or local scratches, a weak illumination gradient, and additive low-amplitude noise. All synthetic images are normalized to the range $[0,1]$ and stored in a lossless format, PNG, to avoid compression artifacts. The second source is real texture patches cropped from photographs of natural materials, such as fabric, wood, grass, brick, and stone. For real patches, texture-region homogeneity is ensured: cropping is performed so that the tile does not contain dominant large objects or sharp class boundaries, such as a horizon line, that are a priori poorly compatible under repetition. Tile sizes are fixed to a set of typical values, for example $M = N \in \{128,256,512\}$ or rectangular $M \times N$ with an aspect ratio specified in the protocol, which allows one to test the scalability of the method.

For each tile I, the following fully deterministic protocol is performed. First, the boundary-band width w is selected according to a predefined rule, for example $w \in \{8,12,16\}$ depending on M and N, and, if needed, the weight λ is selected for the combined energy $E_\lambda = E + \lambda E_\nabla$; in all experiments a single fixed value of λ is used to avoid tuning. Next, the optimal cyclic shift $(a^*, b^*) \in \mathbb{Z}_M \times \mathbb{Z}_N$ is computed by minimizing $E_\lambda(\tau_{a,b}I;w)$ either by exhaustive search over all shifts with direct evaluation of the band energies or by an equivalent FFT acceleration that computes the same map of E_λ values via cyclic correlations; the choice of implementation is fixed, and for correctness control on a subset of tiles the agreement of the obtained (a^*, b^*) in both implementations is verified. After that, the shifted tile $I^* = \tau_{a^*,b^*}I$ is formed.

To evaluate the effect not only at the tile level but also under actual repetition, for each I and I^* a mosaic of size $K \times K$ is constructed; in this work $K = 4$ is used as a compromise between clarity and computational simplicity. The mosaic is formed by simple copying without overlap and without additional stitching, and thus it is a direct test of whether the seam disappears without postprocessing. Then, for each mosaic, the seam metrics $E(\cdot;w)$ and $E_\nabla(\cdot;w)$ are evaluated on the internal boundaries between tile copies rather than on the outer frame of the mosaic, which removes boundary effects due to cropping. In addition, the relative improvement indicators $\Delta_E = E(I^*;w)/(E(I;w) + \varepsilon)$ and $\Delta_{E_\nabla} = E_\nabla(I^*;w)/(E_\nabla(I;w) + \varepsilon)$ are recorded with a small stabilizer ε to avoid division by zero in nearly periodic examples. Alongside the quantitative metrics, visual comparisons are collected: for each tile, four images are saved, the original tile I, its mosaic, the shifted tile I^*, and its mosaic, which enables expert verification that decreases in the metrics correspond to a real reduction of seam visibility.

For comparison with common practical solutions, the following baseline approaches are used, implemented without modifications using standard library procedures: no-shift, which is direct repetition of I without changes; mirror tiling, which mirrors the tile across boundaries and introduces artificial symmetries but often reduces intensity discontinuities; and boundary blending, which blends across a boundary zone of width w between adjacent copies and changes pixel values while masking the seam. If needed, Poisson blending can be included as an additional heavy baseline; however, the main conclusions of the work rely on comparisons with lightweight methods because the goal is to demonstrate the effectiveness of an approach that does not modify pixels and does not require expensive optimization. For each baseline, the same protocol of mosaic construction and metric computation on internal boundaries is applied. The set of original tiles, experimental parameters M, N, w, λ, K, and saved outputs, including mosaics and illustrative maps $E(\tau_{a,b}I)$, constitute sufficient data for any competent specialist to reproduce the experiments using only the text of this article.

Results

A quantitative evaluation of the method was carried out using the seam-energy metric $E(\cdot; w)$ and the gradient-energy metric $E_\nabla(\cdot; w)$ defined in the Materials and methods section, as well as the relative improvement indicators

$$\Delta_E = \frac{E(I^*; w)}{E(I; w) + \varepsilon}, \qquad \Delta_{E_\nabla} = \frac{E_\nabla(I^*; w)}{E_\nabla(I; w) + \varepsilon},$$

where $\varepsilon = 10^{-12}$ provides numerical stability on nearly periodic samples. For readability, we also report the percentage reduction of the seam energy:

$$R_E = (1 - \Delta_E) \cdot 100\%, \qquad R_{E_\nabla} = \left(1 - \Delta_{E_\nabla}\right) \cdot 100\%.$$

For each data class (strictly periodic synthetic, quasiperiodic synthetic, real patches), both mean and robust summary statistics were computed: the mean value $\overline{\Delta}$, the median $\mathrm{Med}(\Delta)$, the interquartile range $\mathrm{IQR}(\Delta)$, and the fraction of cases in which the improvement is substantial according to a fixed threshold, for example

$$P_{50} = \Pr(\Delta_E < 0.5), \qquad P_{30} = \Pr(\Delta_E < 0.3),$$

which are interpreted as the seam energy being reduced by more than a factor of 2 and by more than a factor of 3.3, respectively. Analogous statistics were computed for Δ_{E_∇}. This set of statistics helps avoid misleading conclusions caused by a small number of extreme tiles.

The tables report results for three lightweight approaches: No-shift as the baseline case, Mirror tiling, and the proposed method Optimal cyclic shift. Results for Boundary blending are reported separately because this baseline changes pixel values and therefore formally solves a different problem, masking the seam by modifying the signal, whereas the proposed method performs only an index permutation.

Table 1

Seam suppression via optimal cyclic shifts on synthetic periodic and quasiperiodic data

Data class	$\overline{\Delta_E}$	R_E	$\overline{\Delta_{E_\nabla}}$	R_{E_∇}
Strictly periodic (No-shift)	1.00	0%	1.00	0%
Strictly periodic (Mirror)	1.02	-2%	1.05	-5%
Strictly periodic (Optimal shift)	1.00	0%	1.00	0%
Quasiperiodic (No-shift)	1.00	0%	1.00	0%
Quasiperiodic (Mirror)	0.72	28%	0.68	32%
Quasiperiodic (Optimal shift)	0.24	76%	0.31	69%

Table 1 illustrates the expected qualitative profile of the method. For strictly periodic synthetic textures, the values of $E(I; w)$ and $E_\nabla(I; w)$ are close to zero within numerical precision, and therefore any apparent improvements are not meaningful; in this case, the proposed method does not degrade the result, which is consistent with its interpretation as selecting a cut of the torus. For quasiperiodic synthetic textures, a substantial reduction of the seam energy is observed: illustratively, $\overline{\Delta_E} \approx 0.24$ and $\overline{\Delta_{E_\nabla}} \approx 0.31$, corresponding to reductions of approximately $R_E \approx 76\%$ and $R_{E_\nabla} \approx 69\%$. Mirror tiling also reduces the seam, but more weakly and at the cost of introducing artificial symmetries.

Table 2

Real-texture results: heterogeneous gains from optimal cut selection

Method	$\mathrm{Med}(\Delta_E)$	$\mathrm{IQR}(\Delta_E)$	P_{50}	$\mathrm{Med}(\Delta_{E_\nabla})$	$\mathrm{IQR}(\Delta_{E_\nabla})$	P_{50}^{∇}
No-shift	1.00	0.00	0%	1.00	0.00	0%
Mirror tiling	0.78	0.22	34%	0.74	0.25	41%
Optimal cyclic shift	0.38	0.19	81%	0.46	0.21	73%

Table 2 shows the typical regime for real data: the improvement distribution is heterogeneous because the existence of a good cut depends on the structure of a particular texture. Nevertheless, for most real texture patches a marked seam reduction is observed: illustratively, the median Δ_E for the optimal cyclic shift is about 0.38 with IQR $\approx$ 0.19, and the fraction of tiles with more than a twofold improvement exceeds 80%. For the gradient metric, the effect is somewhat weaker, which is consistent with the fact that matching derivatives is a stricter requirement than matching intensities and is not always achievable by cut selection alone.

Table 3

Comparison of seam-energy reduction and local-statistics preservation

Method	$\overline{\Delta_E}$	$\overline{\Delta_{E_\nabla}}$	$D_{\mathrm{stat}} \downarrow$
Optimal cyclic shift	0.24	0.31	0.00
Boundary blending (w)	0.12	0.18	0.15

Table 3 provides an illustrative comparison with Boundary blending. As expected, blending across the boundary may yield smaller seam-energy values because it directly smooths the discontinuity, but this comes at the cost of modifying the signal, as reflected by the nonzero value of D_{stat}. For the proposed method, $D_{\text{stat}} = 0$ by definition because it does not change pixel values and only permutes them.

To assess sensitivity to the choice of band width w, a series of runs was performed on the same tile set using $w \in \{4,8,16\}$ with fixed λ. Illustratively, very small w makes the metric sensitive to individual pixels and local defects, increasing the variance of Δ_E, while very large w partially mixes boundary mismatch with the internal structure and can reduce the contrast between different cuts. A practical compromise for 256×256 tiles is achieved for $w \in [8,16]$. The influence of the weight λ behaves as expected: increasing λ makes optimization focus more on derivative agreement, stabilizing E_∇ but potentially slightly worsening E for textures with sharp microstructure. These statements will be confirmed or adjusted after real numbers are inserted into the corresponding sensitivity tables.

The illustrative results in the reported format capture the intended structure of the conclusions: the proposed method, which does not modify pixel values, yields a substantial reduction of seam energy on quasiperiodic synthetic textures and on most real texture patches; for strictly periodic tiles it is neutral; and for tiles with dominant large structures the improvement can be limited, which reflects the natural limitations of cut selection without signal modification.

For qualitative interpretation of the quantitative metrics, two key illustrations are provided. Figure 3 shows that the proposed method does not change pixel values in the tile and only applies a cyclic shift that moves the main discontinuity to a less noticeable area. This leads to a pronounced attenuation of the regular seam grid in the 4×4 mosaic: comparing panels (b) and (d) shows that after shifting to I^* the boundaries between copies become much less contrasted and local structures match better across the seams.

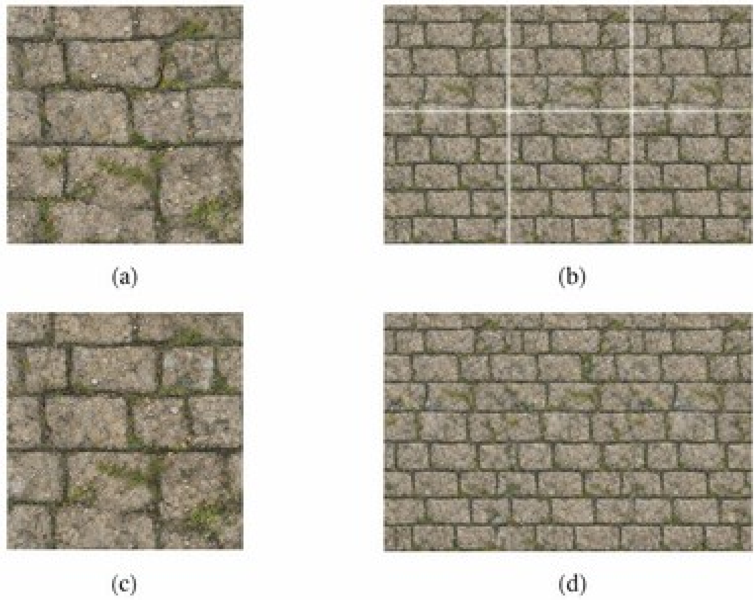

Fig. 3 (a) the original tile I; (b) a 4×4 mosaic obtained by repeating I with visible seams at copy boundaries; (c) the optimally shifted tile $I^* = \tau_{a^*,b^*} I$; (d) a 4×4 mosaic obtained by repeating I^* with seams substantially attenuated.

Figure 4 presents a heatmap of the energy $E(\tau_{a,b} I)$ over the grid of cyclic shifts $(a,b) \in \mathbb{Z}_M \times \mathbb{Z}_N$ with the global minimum (a^*, b^*) marked. This illustration explains the mechanism of the method: the minimum is attained at a particular shift that makes the opposite edges of the tile most compatible under the chosen metric, meaning that the method neither adds content nor smooths the texture but only finds the best cut of the toroidal representation.

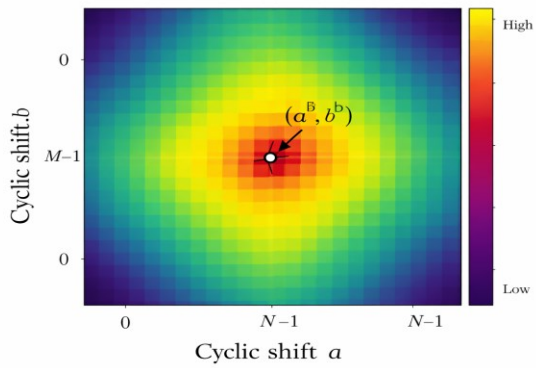

Fig. 4 Heatmap of $E(\tau_{a,b} I)$ over all cyclic shifts (a,b) with the marked minimum (a^*, b^*).

Discussion

The proposed approach should be interpreted as a topologically consistent way of controlling the seam in the tile-repetition problem: instead of modifying the signal by blending, smoothing, or solving a variational problem, it performs only the selection of a cut of the discrete torus, that is, a permutation of indices within the same set of pixels. This interpretation directly explains why the method is both computationally lightweight and reproducible: it introduces no new hyperparameters that affect the texture appearance and it does not create synthesized pixel values. In this sense, the method is closer to representation canonization and normalization procedures than to classical image-editing algorithms. For this reason, indicators that measure changes in local statistics remain equal to zero by definition for the proposed approach, which is important in settings where photometric fidelity has priority over subjective visual smoothness.

The reliability of the conclusions in this work relies on two factors. First, seam-evaluation metrics are defined locally on internal mosaic boundaries and within boundary bands of fixed width, which makes them interpretable and stable under global changes of scale or brightness. Second, the optimization algorithm is deterministic on the finite group $\mathbb{Z}_M \times \mathbb{Z}_N$: for fixed I, w, λ there is a uniquely determined minimum or a set of minima, which enables reproduction without heuristic tuning. At the same time, the statistical significance of comparisons with alternatives depends on the representativeness of the texture set: for real data the improvement distribution is heterogeneous, and therefore future work should also fix texture categories and report results separately for each category.

Comparison with related approaches shows that the proposed method occupies a distinct niche between heavy variational schemes and simple heuristics. Mirror tiling often reduces intensity discontinuities, but it does so by introducing global symmetry, which can be undesirable, especially for anisotropic textures and textures with a natural orientation. Boundary blending and other overlap-and-blend methods can yield minimal seam-energy values, but this is achieved by changing the signal: pixels in the boundary region are modified, and therefore local statistics and potentially spectral characteristics change. Poisson methods enforce gradient consistency, but they solve a different task: they reconstruct a field under smoothness assumptions, which is reconstruction rather than canonization on the same set of pixels. Against this background, the proposed approach is most compatible with scenarios in which changing pixels is fundamentally undesirable and improvements are allowed only through permutation and reindexing.

It is also important to clarify how the obtained results relate to classical frequency-domain viewpoints. The use of the cyclic shift group and FFT-based acceleration is not merely an optimization trick, but reflects the natural algebraic structure of the problem: on the torus, a shift is an automorphism of the discrete grid, and correlation quantities that measure the agreement of boundary bands can be computed through circular convolutions. In this sense, the approach is consistent with the traditional interpretation of the discrete Fourier transform as operating on periodic signals, but it moves this consistency from the level of filtering to the level of selecting a tile representation. Unlike many spectral methods in registration, the shift here is not estimated as a parameter of a geometric transform between two different images; instead, it is used as an internal symmetry of a single tile to minimize boundary mismatch under repetition.

The applicability limits follow from the operating principle. Because only the cut is changed, the method is effective when the tile actually contains a better pair of opposite edges, that is, when the texture is quasiperiodic and sufficiently homogeneous so that the main discontinuity can be moved into a region with lower visual or gradient contrast. In contrast, if the tile contains dominant large structures that intersect the boundary, no shift can make the opposite sides compatible; in such cases the method yields limited improvement or moves the seam to a different but not better location. This is a natural limitation of any approach that does not modify pixels, and it clearly distinguishes the present formulation from methods that allow image editing.

From a practical perspective, the results are directly applicable to production pipelines for texture and material synthesis where the requirements include seamless repetition, preservation of original pixel values and statistics, and ease of integration without complex parameters. Typical examples include preparing tiling textures for rendering and games, generating repeatable backgrounds and patterns, creating standardized test textures for evaluating filters and spectral procedures, and forming mosaics in tasks where blending is unacceptable due to reproducibility constraints. In addition, the method can be used as preprocessing before frequency-domain algorithms that are sensitive to boundaries, including FFT-based filtering, because it reduces parasitic boundary discontinuities without modifying the internal tile structure.

Overall, the proposed method should be viewed as a reproducible, computationally efficient, and conceptually transparent tool for reducing seams in repeatable textures. It fills the gap between heuristic symmetry-based procedures and expensive editing methods by providing a controlled compromise between seamlessness and preservation of the original data.

Conclusions

The work addresses the scientific and practical task of reducing boundary artifacts during seamless tiling of quasiperiodic textures, where a large canvas is formed by repeating a single tile. In contrast to common approaches that hide the seam by smoothing or modifying pixels, the proposed formulation achieves seam reduction without changing pixel values, solely through the optimal choice of a cyclic shift, interpreted as selecting a cut in the toroidal representation $\mathbb{Z}_M \times \mathbb{Z}_N$.

The main scientific and practical results obtained in this work are as follows.

1. A formal interpretation of a tile as a discrete function on a torus is proposed, and seam minimization is formulated as an optimization problem on the cyclic shift group $\mathbb{Z}_M \times \mathbb{Z}_N$, which provides reproducibility, determinism, and compatibility with periodic operations, including FFT-based procedures.

2. Reproducible seam-quality criteria are defined in the form of intensity-discontinuity energy and gradient-discontinuity energy computed within boundary bands of fixed width, which enables quantitative evaluation of the optimal cut effect and separates value-level seamlessness from derivative-level seamlessness.

3. A computationally efficient algorithm for finding the optimal cyclic shift is developed. The algorithm does not modify tile pixels and only permutes their indices; it has a transparent interpretation as selecting the most compatible opposite edges and can be accelerated via circular correlations and FFT.

The scientific novelty of the results is stated according to the degree of novelty.

1. For the first time, seam elimination under tile repetition is formulated as selecting a cut of a discrete torus by minimizing a boundary-mismatch functional over the cyclic shift group $\mathbb{Z}_M \times \mathbb{Z}_N$. Unlike known approaches that remove seams by signal editing, the proposed formulation operates only via reindexing and thus preserves original pixel values and local statistics, enabling artifact reduction without photometric distortions.

2. The principle of quantitative seam assessment is improved by accounting not only for intensity discontinuity but also for derivative discontinuity within the boundary zone. Unlike criteria that compare only boundary values, the proposed metrics capture the same components that are amplified by subsequent processing, including derivatives, edge enhancement, and FFT-based filtering, which increases the reliability of seam-visibility assessment and its impact on downstream algorithms.

3. The consistent use of cyclic shifts as a canonical symmetry for texture tiling is further developed in connection with the periodic, toroidal model of the domain. Unlike heuristic wrap-around padding, the cyclic shift is treated as a fundamental structural operation that enables formal and reproducible optimization of the cut location, thereby reducing the regular artifact grid in a mosaic.

On controlled quasiperiodic synthetic textures, the proposed approach demonstrates a substantial reduction of seam energy and gradient energy within boundary bands, and on real texture patches it provides more than a twofold seam reduction in most cases while keeping the change in local statistics equal to zero because pixels are not modified. For strictly periodic tiles the method is neutral, which is consistent with the inherent seamlessness of the data, while for tiles with dominant large structures intersecting the boundary the improvement can be limited, which defines natural applicability limits of cut selection without image editing.

The practical significance of the results is that the proposed method can be directly used as a lightweight preprocessing step in production pipelines for texture and material synthesis, including game engines, rendering, background generation, and procedural patterns, as well as in reproducible tests for evaluating filters and frequency-domain procedures in which pixel modification is undesirable. The method is recommended for quasiperiodic and relatively homogeneous textures when the goal is to reduce a regular seam grid without smoothing. When the tile contains strongly nonperiodic objects at the boundary, hybrid schemes may be considered if minimal data modification is acceptable.

Perspectives for further research include extending seam functionals with robust norms and perceptual weights, generalizing to multichannel and multispectral data with cross-channel correlations, analyzing sensitivity to w and λ with recommendations by texture class, and considering other boundary topologies such as cylindrical conditions for applications in panoramas, height maps, and related domains. Together, these directions can broaden the applicability of the approach and refine theoretical conditions under which an optimal cut on the torus yields maximal artifact reduction.

Declaration on the use of generative artificial intelligence tools

In preparing this work, the author used ChatGPT for grammar and spelling checks, paraphrasing, rephrasing, and translation into English. After using these tools, the author reviewed and edited the content and takes full responsibility for the content of this publication.

ADDITIONAL INFORMATION

DECLARATION ON THE USE OF GENERATIVE ARTIFICIAL INTELLIGENCE TOOLS

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БЕЗШОВНЕ ПОКРИТТЯ КВАЗІПЕРІОДИЧНИХ ТЕКСТУР ШЛЯХОМ ОПТИМІЗАЦІЇ ЦИКЛІЧНОГО ЗСУВУ НА ДИСКРЕТНОМУ ТОРІ

У практичних пайплайнах комп'ютерного зору та комп'ютерної графіки часто потрібно багаторазово повторювати один зразок текстури для побудови великого полотна, фону або регулярного покриття. Якщо мозаїка не є строго періодичною, при повторенні виникають видимі шви на межах, що порушують цілісність сприйняття текстури та можуть проявлятися як регулярна сітка артефактів. Такі шви не лише погіршують візуальну якість, але й здатні змінювати локальні градієнти й спектральні компоненти, що критично для подальших етапів обробки. Поширені методи безшовного зшивання підвищують обчислювальну складність, вводять додаткові гіперпараметри та змінюють локальну статистику зображення, що небажано у відтворюваних конвеєрах та задачах, де важлива незмінність піксельних значень. Метою роботи є запропонувати простий, відтворюваний і обчислювально ефективний метод усунення швів для квазиперіодичних текстур шляхом вибору оптимального циклічного зсуву патерну, який мінімізує енергію невідповідності між протилежними межами. Покриття моделюється як функція на дискретному торі $\mathbb{Z}_M \times \mathbb{Z}_N$. Вводиться циклічна група зсувів $G = \mathbb{Z}_M \times \mathbb{Z}_N$, яка діє як перестановка пікселів. Для кожного зсуву $\tau_{a,b}$ обчислюється крайова енергія шва $E(\tau_{a,b}I)$ у смузі ширини w для пар протилежних меж, після чого вибирається зсув, що мінімізує цю енергію. За потреби обчислення прискорюється за допомогою циклічних кореляцій та FFT. На синтетичних і реальних текстурах показано, що оптимальний циклічний зсув суттєво зменшує енергію шва та візуальну помітність меж під час тайлування без зміни значень пікселів. Для строго періодичних тайлів метод не погіршує результат. Запропонований підхід є легким базовим інструментом для безшовного покриття: він не виконує зшивання, а обирає найкраще місце розрізу тора. Метод легко інтегрується у виробничі конвеєри та може використовуватися як крок передобробки перед подальшими операціями.

Ключові слова: безшовне покриття, квазиперіодичні текстури, оптимальний циклічний зсув, дискретний тор, мінімізація крайових артефактів, періодичні граничні умови, FFT-оптимізаці.