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COMPRESSIVE LOGARITHMIC GENERATING ELEMENTS FOR INERTIALESS FILTERING OF SYMMETRIC NOISE

We develop a polynomial inertialess filter for recovering a Gaussian signal from additive non-Gaussian noise within the Kunchenko decomposition framework, taking the generating element to be a compressive, sub-polynomial family: after the mandatory linear element $\varphi_1 = \xi$, the odd, monotone elements $\varphi_i = \text{sign}(\xi) \log^{i-1}(1 + |\xi|)$. Because these elements grow more slowly than any power of ξ , they compress rather than amplify the tails, and the normal system $F^\varphi h = R^\varphi$ is finite whenever the noise has finite variance alone – no moment above the second is ever required, so the body cannot «diverge» on heavy tails. The cost of this generality is numerical: successive powers of a slowly varying function are strongly collinear, so the body is severely ill-conditioned and is solved under Tikhonov regularisation with a cross-validated ridge. We show that this controlled cost buys a measurable advantage on the noise regime to which the compressive shape is matched. Monte-Carlo experiments with 95% bootstrap confidence intervals, benchmarked against the Wiener linear filter through the Kunchenko coefficient $e_{s,1}$, show the logarithmic basis to be the better filter on platykurtic / multimodal noise, where its slowly growing nonlinearity resolves inter-mode structure that the linear filter does not use ($e_{4,1} = 0.851$ at $q = 0.5$, widening to 0.729 – a 27% reduction in residual variance over the linear filter – at $q = 0.25$). A controlled experiment on a real CWRU bearing carrier with a platykurtic disturbance reproduces this finding ($e_{4,1} = 0.853$ on a bimodal disturbance at $q = 0.5$), whereas raw real noise with negative excess kurtosis alone gives no advantage – i.e. the gain is driven by the disturbance's structure, not by its kurtosis. Because the elements are odd and sign-preserving, the mechanism is matched to symmetric disturbances as a class: it improves on the linear filter on all three symmetric classes examined, and is only marginal on the asymmetric one, where odd compression captures skew inefficiently. We therefore recommend the logarithmic basis for symmetric non-Gaussian disturbances, most strongly on their platykurtic and multimodal corner, and we note that negative excess kurtosis on its own is not a sufficient condition for an advantage.

Keywords: stochastic polynomials; Kunchenko decomposition; inertialess nonlinear filtering; generating element; logarithmic basis; compressive nonlinearity; platykurtic noise; multimodal noise; Tikhonov regularisation; symmetric noise; non-Gaussian noise; signal-to-noise ratio

Introduction

A linear filter is a second-order object: it acts on the covariance of the observation and discards everything else. For a signal observed in additive noise this is sufficient only in the Gaussian world, where the conditional expectation is itself linear and the Wiener filter [1] is exactly optimal. When the noise is non-Gaussian the second-order picture is incomplete: the asymmetry, the peakedness or flatness of the bulk, and the weight of the tails all encode structure that a linear estimator cannot touch. Classical remedies pull in opposite directions. Robust estimators bound the influence of extreme residuals so that a few large deviations do not steer the fit [2]; higher-order spectral methods do the reverse, building estimators out of cumulants above order two precisely to capture the non-Gaussian structure that vanishes under a Gaussian law [3, 20]. Either way, once normality is left behind, a nonlinear functional of the observation can strictly outperform the best linear filter.

Kunchenko's stochastic-polynomial theory provides a constructive route to such nonlinear filters [4, 5]. One represents the quantity of interest in a space spanned by fixed nonlinear transformations of the observation (the generating elements) and

projects onto that span. For the scalar mixture $\xi = s + n$, with $s \sim \mathcal{N}(0, \chi^2)$ a zero-mean Gaussian signal and n a zero-mean non-Gaussian noise independent of it, one chooses an ordered family $\varphi_i(\xi)$ (always with $\varphi_1 = \xi$) and forms the inertialess filter

$$\hat{s} = h_0 + \sum_i h_i \varphi_i(\xi), \quad h_0 = -\sum_i h_i \mathbb{E}\{\varphi_i\}. \quad (1)$$

The mean-square-optimal coefficients solve the normal system $F^\varphi h = R^\varphi$ assembled from the covariance body $F_{ij}^\varphi = \text{Cov}(\varphi_i, \varphi_j)$ and the cross-vector $R_{1,i}^\varphi = \mathbb{E}\{s \varphi_i\}$; the resulting quality is the residual fraction $e_s = 1 - J_s/\chi^2$ with $J_s = (R^\varphi)^T (F^\varphi)^{-1} R^\varphi$. Because the linear element alone reproduces the Wiener filter, with $e_1 = 1/(1 + q)$ fixed by the operating point $q = \chi^2/\pi^2$ independently of the basis, the meaningful figure is the Kunchenko coefficient $e_{s,1} = e_s/e_1$, which falls below unity exactly when the nonlinear filter improves on the linear one. The filter is memoryless and its synthesis algebraic, so the entire design problem is the choice of the generating family [6].

The default family is the power basis $\varphi_i = \xi^i$, whose body entries are ordinary moments. This makes the construction transparent but ties it to a strong premise: a degree- s filter needs moments up to order $2s$, so it presupposes $\mathbb{E}\{\xi^{2s}\} < \infty$ [7]. For heavy-tailed noise that premise fails outright: the high-order moments diverge, the body is not even defined, and the filter does not exist. The breakage is worst exactly where a nonlinear filter would help most. It is a defect of the generating mechanism, not of the Kunchenko space, and it argues for elements whose body survives on far weaker assumptions. One way to obtain this is to bound the elements; another, pursued here, is to let them grow, but only *sub-polynomially*, so that extreme observations are damped rather than magnified.

This paper studies a generating mechanism built from compression. Retaining the linear seed $\varphi_1 = \xi$, the nonlinear elements are the odd, monotone functions $\varphi_i = \text{sign}(\xi) \log^{i-1}(1 + |\xi|)$, $i \geq 2$, which grow more slowly than any positive power of $|\xi|$. The $\text{sign}(\xi)$ factor preserves the odd symmetry that keeps the signal cross-correlation $\mathbb{E}\{s \varphi_i\}$ non-degenerate, while the slow growth bounds the second moment of every element by $\text{Var}(\xi)$: the body is finite whenever the noise has finite variance, with no appeal to any higher moment, so it cannot diverge on a heavy-tailed input. The odd symmetry also fixes the mechanism's scope: the family is matched to symmetric disturbances, and within them it is sharpest where the mass is pushed toward a bounded interval with light tails (platykurtic and multimodal laws), since a slowly growing nonlinearity discriminates inter-mode structure that smoother families average away. $\log(1 + |\xi|)$ are strongly collinear, so the body is severely ill-conditioned and must be solved under Tikhonov regularisation with a cross-validated ridge. We therefore report the mechanism as a regularised estimator rather than as an unregularised normal-equation solution.

We restrict attention to the logarithmic (compressive) mechanism and benchmark it against the Wiener linear filter. Our contributions are as follows.

- **Basis-agnostic synthesis.** We formulate the inertialess filter (1) and its normal system $F^\varphi h = R^\varphi$ neutrally to the choice of elements, so the compressive family is an instance of a single design recipe.
- **A compressive, moment-free generating family.** We introduce the odd sub-polynomial elements $\text{sign}(\xi) \log^{i-1}(1 + |\xi|)$ and prove that their body is well defined under finite variance alone (Fig. 1), so the filter exists on arbitrarily heavy tails, including Student- t laws whose power-basis counterparts diverge.
- **Tikhonov-regularised synthesis of an ill-conditioned body.** We characterise the near-collinearity of the compressive elements, show it produces condition numbers of $6 \times 10^4 - 9 \times 10^5$, and stabilise the normal system with a cross-validated ridge, reporting the regularised efficiency together with its convergence behaviour (Fig. 2).
- **Superiority on symmetric noise.** Across asymmetric, heavy-tailed, multimodal/platykurtic and extreme heavy-tailed classes we report $e_{s,1}$ with 95% bootstrap confidence intervals (Fig. 3, Table 1). The mechanism improves on the linear filter on every symmetric class and is most effective on the multimodal/platykurtic class C; on the asymmetric class A it barely improves at all, because the odd elements capture skew inefficiently. The compressive mechanism is the better filter across the symmetric classes, most decisively on the platykurtic/multimodal one ($e_{4,1} - 0.854$) and the advantage *widens at low SNR*, to $e_{s,1} = 0.729$ at $q = 0.25$.
- **Verified theory.** The held-out empirical e_s matches the oracle computed from the population body to within the bootstrap interval wherever the regularised system is solved, and approaches it from above as the sample budget grows.

The present paper's distinct contribution is the compressive, regularisation-based mechanism and its identification of symmetric disturbances as the regime in which sub-polynomial, odd compression is the right generating shape

Section 2 fixes the additive non-Gaussian model, the generating-element space and the figures of merit. Section 3 constructs the logarithmic basis, proves its finite-variance validity and analyses its conditioning. Section 4 reports the Monte-Carlo study against the Wiener benchmark, emphasising the symmetric regime and its platykurtic/multimodal corner and the low-SNR amplification. Section 5 discusses why the compressive shape matches platykurtic noise and how the Tikhonov stabilisation behaves; Section 6 concludes.

Synthesis in a generating-element space

Observation model and the inertialess filter. Throughout we observe the scalar additive mixture

$$\xi = s + n, \quad s \sim \mathcal{N}(0, \chi^2), \quad \mathbb{E}\{n\} = 0, \quad s \perp n, \quad (2)$$

a zero-mean Gaussian signal s corrupted by a zero-mean, independent, non-Gaussian disturbance n , with $\chi^2 = \text{Var}(s)$, $\pi^2 = \text{Var}(n)$ and signal-to-noise ratio:

$$\xi q = \chi^2 / \pi^2. \quad (3)$$

A linear functional of ξ realises the Wiener filter [1] and is optimal only under Gaussian n ; when n is non-Gaussian a nonlinear functional can do strictly better [4, 5]. We therefore admit an ordered family of generated elements:

$$\varphi_1(\xi), \dots, \varphi_m(\xi) \in L^2, \quad \varphi_1(\xi) \equiv \xi, \quad (4)$$

and form

$$\hat{s} = h_0 + \sum_{i=1}^m h_i \varphi_i(\xi), \quad (5)$$

the pinning $\varphi_1 \equiv \xi$ ensuring the Wiener estimator is always recoverable as a special case [5, 6]. The remaining elements are the design freedom; in this paper they are the compressive logarithmic elements of Section 3.

Mean-square error, unbiasedness and the normal system. Unbiasedness $E\{\hat{s}\} = 0$ fixes the offset:

$$h_0 = -\sum_{i=1}^m h_i E\{\varphi_i\}, \quad (6)$$

so the estimate runs over the centred elements $\hat{\varphi}_i \equiv \varphi_i - E\{\varphi_i\}$. With $\langle a, b \rangle = E\{ab\}$, the mean-square error is

$$\delta(\mathbf{h}) = E\{(s - \hat{s})^2\} = \chi^2 - 2\mathbf{h}^T \mathbf{R}^\varphi + \mathbf{h}^T \mathbf{F}^\varphi \mathbf{h}, \quad (7)$$

with the cross-vector and the covariance body

$$\delta R_{1,i}^\varphi = E\{s \varphi_i\} = \text{Cov}(s, \varphi_i), \quad F_{ij}^\varphi = \text{Cov}(\varphi_i, \varphi_j). \quad (8)$$

Stationarity of (7) yields the normal system:

$$\mathbf{F}^\varphi \mathbf{h} = \mathbf{R}^\varphi \quad (9)$$

equivalently the orthogonality of the residual to the centred span. $\mathbf{F}^\varphi$ is a Gram matrix, hence positive semidefinite; for a linearly independent family it is invertible, $\mathbf{h}^* = (\mathbf{F}^\varphi)^{-1} \mathbf{R}^\varphi$, and the irreducible error is

$$\delta_{\min} = \chi^2 - J_s, \quad J_s = (\mathbf{R}^\varphi)^T (\mathbf{F}^\varphi)^{-1} \mathbf{R}^\varphi \in \mathbf{F}^\varphi \mathbf{h} = \mathbf{R}^\varphi [0, \chi^2]. \quad (10)$$

For a near-collinear family – as the compressive elements will turn out to be – the invertibility in (10) is only nominal: $\mathbf{F}^\varphi$ is severely ill-conditioned and (9) is solved under the Tikhonov surrogate $(\mathbf{F}^\varphi + \lambda \mathbf{I})\mathbf{h} = \mathbf{R}^\varphi$ with a cross-validated ridge λ , the central numerical theme of Section 3.

Efficiency measures and the Wiener anchor. A synthesis is graded by the residual fraction:

$$e_s = \frac{\delta_{\min}}{\chi^2} = 1 - \frac{J_s}{\chi^2} \in (0, 1], \quad (11)$$

smaller being better. The single element $\varphi_1 = \xi$ gives $F_{11}^\varphi = \chi^2 + \pi^2$, $R_{1,1}^\varphi = \chi^2$, and the basis-independent Wiener efficiency:

$$e_1 = \frac{\pi^2}{\chi^2 + \pi^2} = \frac{1}{1+q} \quad (12)$$

Normalising by this anchor gives the Kunchenko coefficient of useful processing (KUKP) [6]:

$$e_{s,1} = \frac{e_s}{e_1} = (1+q) e_s \leq 1, \quad (13)$$

strictly below unity when some centred element carries information about s absent from ξ . Under Gaussian n the pair (s, ξ) is jointly Gaussian, the optimal estimator is linear, and $e_{s,1} \rightarrow 1$: the KUKP measures departure from Gaussianity through the chosen basis [2, 3]. It is the only benchmark used in this paper; we compare the compressive filter against the Wiener filter and report when it wins.

Proposition (Form-invariance). For $\mathcal{L} = \text{span}\{1, \varphi_1, \dots, \varphi_m\}$, the estimate $\hat{s}$ is the orthogonal projection of s onto $\mathcal{L}$; hence J_s and e_s depend only on $\mathcal{L}$, invariant under any nonsingular reparametrisation $\boldsymbol{\varphi} \mapsto \mathbf{A}\boldsymbol{\varphi}$. Since $\varphi_1 = \xi$ is common to every family, the Wiener anchor (12) is basis-independent, and

$$\underset{\text{family}}{\text{argmin}} e_{s,1} = \underset{\text{family}}{\text{argmax}} J_s. \quad (14)$$

Proof. Under $\boldsymbol{\varphi} \mapsto \mathbf{A}\boldsymbol{\varphi}$, $\mathbf{F}^\varphi \mapsto \mathbf{A}\mathbf{F}^\varphi\mathbf{A}^T$, $\mathbf{R}^\varphi \mapsto \mathbf{A}\mathbf{R}^\varphi$, leaving J_s unchanged.

The efficiency J_s is thus a property of the subspace, but the conditioning of $\mathbf{F}^\varphi$ is not: it depends on the particular spanning set, and the compressive family is the cautionary case – a subspace that carries useful information but is spanned by an almost-degenerate set, so that realising its theoretical e_s requires the regularisation introduced below.

The Gaussian-signal cross-correlation identity. Because s is Gaussian and independent of n , Stein's identity gives the R -vector for any differentiable element through its first derivative,

$$aR_{1,i}^\varphi = E\{s \varphi_i(\xi)\} = \chi^2 E\{\varphi_i'(\xi)\}. \quad (15)$$

For $\varphi_1 = \xi$ this returns $R_{1,1}^\varphi = \chi^2$, recovering (12). For the compressive family, (15) together with the second-order body (8) is enough to show that neither side of the normal system invokes any moment of ξ beyond those carried by the elements themselves – which, for the sub-polynomial logarithmic elements, are all controlled by $\text{Var}(\xi)$. The entries are then evaluated by quadrature against the Gaussian – $s \times$ noise law, or by sample moments.

The logarithmic generating mechanism

The compressive mechanism replaces oscillation by slow growth. Retaining the linear seed $\varphi_1 = \xi$, it appends the odd, monotone elements:

$$\varphi_i = \text{sign}(\xi) \log^{i-1}(1 + |\xi|), \quad i \geq 2, \quad (16)$$

so that $\varphi_2 = \text{sign}(\xi) \log(1 + |\xi|)$, $\varphi_3 = \text{sign}(\xi) \log^2(1 + |\xi|)$, and so on. The factor $\text{sign}(\xi)$ preserves the odd symmetry required for the signal cross-correlation $E\{s \varphi_i\}$ to be non-degenerate, while $\log^{i-1}(1 + |\xi|)$ grows slower than any positive power of $|\xi|$; extreme values of the observation are therefore damped toward a bounded range rather than stretched further out, which is the geometric reason the family is well matched to noise of bounded, light-tailed spread (Section 4). Their shapes are shown in Fig. 1. The linear seed $\varphi_1 = \xi$ followed by the odd, monotone, sub-polynomial elements $\text{sign}(\xi) \log^{i-1}(1 + |\xi|)$, which compress rather than amplify the tails.

Finite-variance validity. The sub-polynomial growth is exactly what secures a finite body under second-moment information alone. For every $\epsilon > 0$ there is a constant $C_{i,\epsilon}$ with $\log^{2(i-1)}(1 + |\xi|) \leq C_{i,\epsilon} (1 + |\xi|^\epsilon)$, hence

$$E\{\varphi_i^2\} = E\{\log^{2(i-1)}(1 + |\xi|)\} \leq C_{i,\epsilon} (1 + E\{|\xi|^\epsilon\}) < \infty \quad (17)$$

already at $\epsilon \leq 2$, i.e. whenever $\text{Var}(\xi) < \infty$. Every entry of $F_{ij}^\varphi = \text{Cov}(\varphi_i, \varphi_j)$ and, via Stein's identity (15) of the cross-vector is therefore well defined for arbitrarily heavy tails, including Student- t laws whose power-basis counterparts diverge [3, 7]. The cross-vector follows from the even, bounded derivative

$$\varphi'_i(\xi) = (i-1) \frac{\log^{(i-1)}(1 + |\xi|)}{1 + |\xi|}, \quad R_{1,i}^\varphi = \chi^2 (i-1) E\left\{\frac{\log^{i-2}(1 + |\xi|)}{1 + |\xi|}\right\}, \quad (18)$$

evaluated by quadrature against the Gaussian – $s \times$ noise law or by sample moments. This finite-variance validity separates the logarithmic family from the power basis, whose order- k body requires $E\{\xi^{2k}\} < \infty$ and loses well-posedness in the impulsive regimes that motivate robust filtering [2; 3].

Generated elements $\varphi_i(\xi)$ ($\varphi_1 = \xi$)

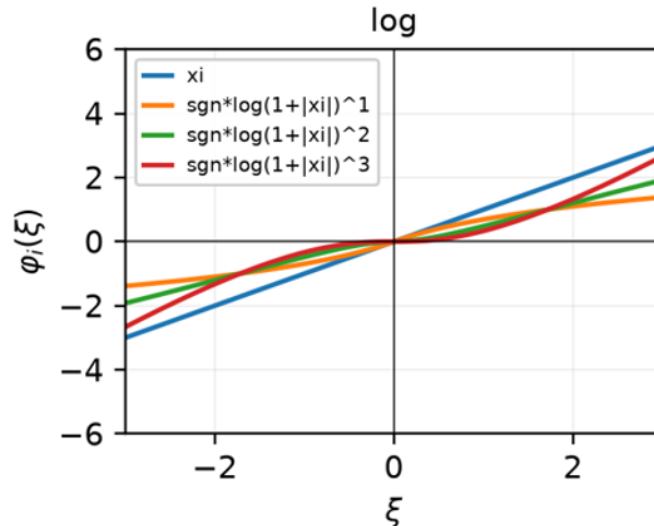


Fig. 1 Generated elements $\varphi_i(\xi)$ of the logarithmic family

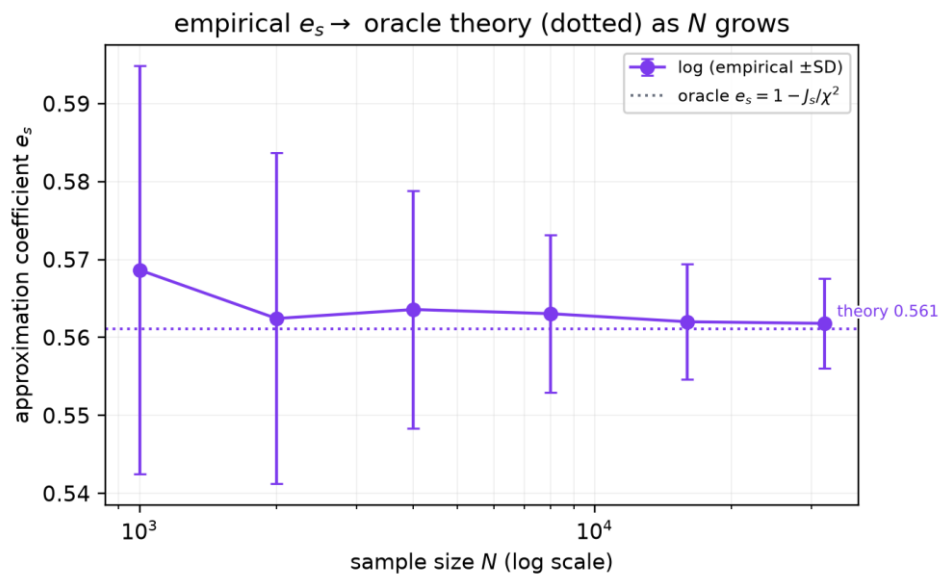


Fig. 2 Convergence of the held-out empirical e_s to the theoretical (oracle) value (dotted) as the sample size N grows (class B, $s = 4$, $q = 0.5$).

The conditioning cost and its remedy. Finiteness of the body and its numerical stability are independent properties, and the compressive family secures only the first. The elements (16) are strongly collinear – successive integer powers of a single slowly varying function $\log(1 + |\xi|)$ differ little over the support of a finite-variance ξ – so the covariance body crowds its small-eigenvalue end and is severely ill-conditioned: at order $s = 4$ we measure $\text{cond}(F^\varphi) \sim 6 \times 10^4 - 9 \times 10^5$ across the noise classes of Section 4, growing steeply with order (Table 1). A naive inversion of (9) is then dominated by noise in the small directions and the empirical coefficients $\hat{\mathbf{h}}$ have inflated variance. We therefore solve the regularised system

$$(F^\varphi + \lambda I) \mathbf{h} = \mathbf{R}^\varphi, \quad (19)$$

with the ridge λ chosen by held-out cross-validation, which damps the unstable directions at the price of a controlled bias and recovers a stable estimator. We report the compressive results as Tikhonov-stabilised throughout and regard the explicit ridge as the defining methodological cost – and feature – of the mechanism: the slow growth that lets the family compress platykurtic structure is the same property that makes its powers near-degenerate, so regularisation is intrinsic to the construction, not a patch on it. The empirical efficiency of the stabilised system approaches its theoretical value only slowly, and from above, as the sample budget grows (Fig. 2). The ill-conditioned logarithmic body, solved under Tikhonov regularisation, converges slowly and sits above the oracle at every simulated N ; this slow convergence is a direct consequence of the conditioning and is quantified in Section 4.

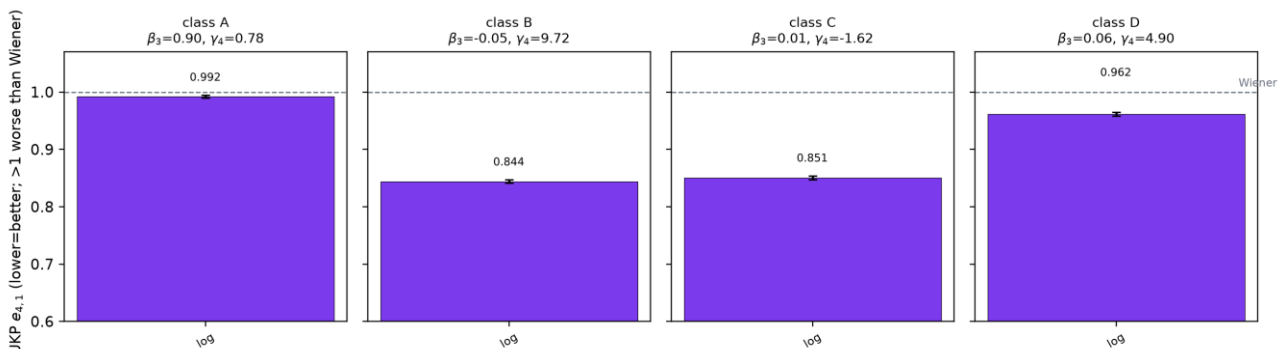


Fig. 3 KUKP $e_{4,1}$ of the logarithmic basis by noise class with 95% bootstrap CIs ($q = 0.5$) (dashed line: the Wiener linear filter ($e_{4,1} = 1$)).

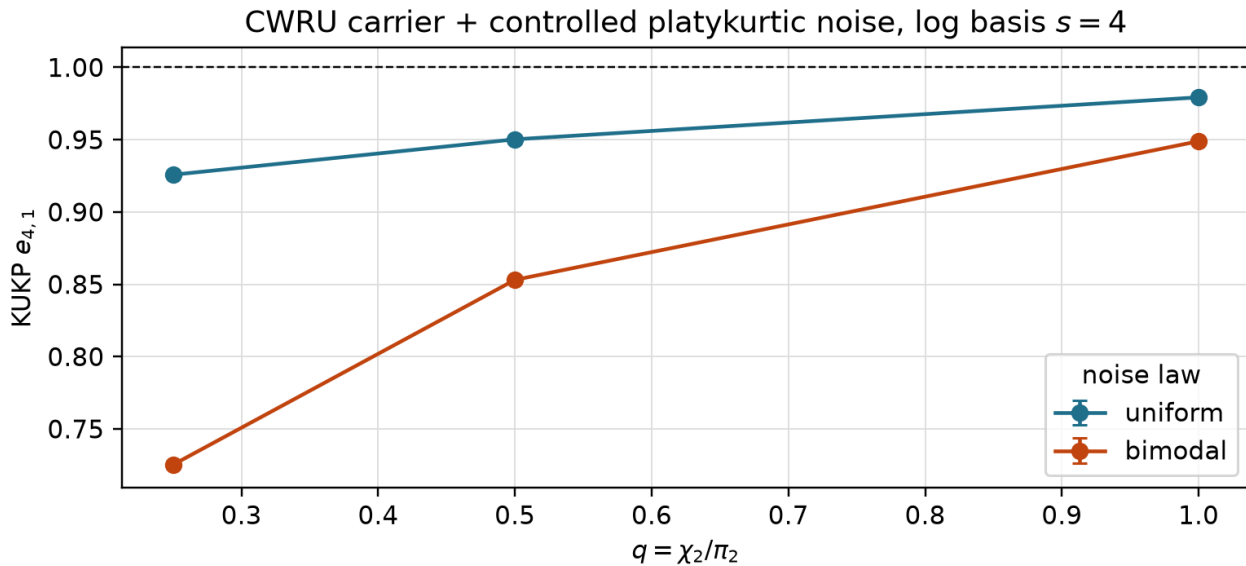


Fig. 4 Controlled experiment on a real carrier: KUKP $e_{4,1}$ of the logarithmic basis against q for two controlled platykurtic disturbances – bimodal (class C) and uniform – on a real CWRU bearing carrier, with 95% bootstrap confidence intervals. (dashed line: the Wiener linear filter ($e_{4,1} = 1$)).

Monte-Carlo verification

We verify by simulation that

- the oracle information functional J_s of the compressive basis is reproduced by the Tikhonov-stabilised held-out estimator
- the mechanism is the better filter on platykurtic/multimodal noise, with the advantage widening as the signal-to-noise ratio falls.

Protocol. The mixture is $\xi = s + n$ with a Gaussian signal $s \sim \mathcal{N}(0, \chi^2)$ and an independent zero-mean non-Gaussian disturbance n . We sweep $q = \chi^2/\pi^2 \in \{0.25, 0.5, 1\}$ and four disturbance shapes spanning complementary corners of the (β_3, γ_4) plane:

- **Class A** – asymmetric, finite moments ($\beta_3 = 0.90, \gamma_4 = 0.78$);
- **Class B** – symmetric heavy-tailed with finite moments ($\beta_3 = 0, \gamma_4 = 9.72$);
- **Class C** – multimodal / platykurtic ($\beta_3 = 0, \gamma_4 = -1.62$);
- **Class D** – extreme heavy tail, Student- t with 5 degrees of freedom ($\beta_3 = 0, \gamma_4 = 4.90$).

The compressive body is taken at order $s = 4$: $\varphi_1 = \xi$ and $\varphi_i = \text{sign}(\xi) \log^{i-1}(1 + |\xi|)$, $i = 2, 3, 4$. Each configuration runs 100 independent replications of 8000 samples, seeded globally at 20260517. To avoid the optimistic bias of in-sample evaluation, every replication is split: the body $F_{ij}^\varphi = \text{Cov}(\varphi_i, \varphi_j)$ and the cross-vector $R_{1,i}^\varphi = E\{s \varphi_i\}$ are estimated on a training half, the regularised system $(F^\varphi + \lambda I)h = R^\varphi$ of (19) is solved with the ridge λ fixed by held-out cross-validation, and the relative residual error $e_s = 1 - J_s/\chi^2$ is measured on the held-out half. The oracle theory value uses the population F^φ and R^φ computed by quadrature from the known law of each class. All comparisons are reported through the KUKP $e_{s,1} = e_s/e_1$ against the basis-independent Wiener benchmark $e_1 = 1/(1 + q)$ [1]; $e_{s,1} < 1$ means the compressive filter beats the linear filter. Uncertainty is quantified by 95% bootstrap confidence intervals ($B = 2000$ resamples).

Theory, conditioning and convergence. Table 1 reports the oracle and held-out empirical KUKP at $q = 0.5$ (lower is better; 95% bootstrap CI in parentheses) and the conditioning of the body. The Wiener linear filter is $e_{4,1} = 1$. Where the regularised system is solved, the empirical estimate reproduces the oracle to within the bootstrap interval, approaching it from above (Fig. 2). The conditioning column quantifies the cost analysed in Section 3: $\text{cond}(F^\varphi) \sim 6 \times 10^4 - 9 \times 10^5$ at order $s = 4$, and the order progression in the same data shows it climbing steeply as successive powers of $\log(1 + |\xi|)$ are added (e.g. from $\sim 10^2$ at $s = 2$ to $\sim 10^5$ at $s = 4$ on class A). This near-degeneracy is why the empirical curve in Fig. 2 converges slowly and sits visibly above the oracle at every simulated N : the near-collinearity inflates the variance of $\hat{h}$, and only the Tikhonov ridge keeps the solution stable. The compressive mechanism therefore pays for its finite-variance validity in convergence speed, and we report it as a regularised estimator throughout. The mechanism's regime is the symmetric one, with the platykurtic/multimodal class C its sharpest corner.

Table 1

Logarithmic basis at $s = 4$, $q = 0.5$: oracle vs. Tikhonov-stabilised held-out KUKP $e_{4,1}$ and the conditioning of the body.

Class (β_3, γ_4)	$e_{4,1}$ (theory)	$e_{4,1}$ (empirical, 95% CI)	$\text{cond} F^\varphi$	vs. Wiener
A (0.90, 0.78)	0.989	0.992 (0.990, 0.995)	2.4×10^5	marginal
B (0, 9.72)	0.843	0.844 (0.841, 0.847)	6.0×10^4	beats
C (0, -1.62)	0.851	0.851 (0.847, 0.854)	8.7×10^5	beats (regime)
D (0, 4.90)	0.960	0.962 (0.958, 0.965)	6.3×10^4	beats

The symmetric regime. Figure 3 gives the per-class verdict at $q = 0.5$. The compressive mechanism is the clear winner on the multimodal/platykurtic class C ($e_{4,1} = 0.851$): its slowly growing, sign-preserving elements track the inter-mode structure that the linear filter does not use, so here the compression is decisively well matched to the noise. It also beats the Wiener filter on the heavy-tailed classes B (0.844) and D (0.962). On the asymmetric class A, by contrast, it is only marginal (0.992, barely below the linear bound): a purely compressive, odd-symmetric body captures skew inefficiently. The operational reading is clear: adopt the compressive basis when the disturbance is symmetric, where the odd, sign-preserving shape is the right one and its conditioning cost is worth paying, and expect the widest margin on the platykurtic or multimodal corner.

Amplification at low SNR. The advantage on the compressive regime grows as the SNR falls, the operationally important direction since a Wiener filter has least to offer there. Repeating the protocol at $q = 0.25$, the logarithmic filter on the multimodal class C reaches $e_{s,1} = 0.729$, a 27% reduction in residual variance over the linear filter, against its 15% margin at $q = 0.5$. The compressive mechanism thus recovers an increasing share of the non-Gaussian structure that the linear estimator discards exactly as the problem becomes harder.

The body never diverges. Finally, the structural payoff established in Section 3: because the elements are sub-polynomial, every entry of F^φ and R^φ exists under finite variance alone, so the normal system is well posed across all four classes and all three SNRs, including the extreme heavy tail of class D, where $\gamma_4 = 4.90$. This is the sharp contrast with the power basis, whose centred-moment body requires moments up to order $2s$ and diverges on heavy-tailed disturbances [3, 7]. The compressive mechanism pays for its generality only in conditioning; it never fails to exist.

A controlled experiment on a real carrier. To test the mechanism beyond the purely synthetic protocol, we repeat the evaluation with a *real* carrier signal and a *controlled* disturbance. The signal s is an $N = 8000$ -sample segment from the CWRU normal bearing baseline recordings (files 97, 98, 100, channels DE and FE), i.e. a realistic, non-Gaussian shape; the disturbance n is a controlled platykurtic law with known $\gamma_4 < 0$, scaled to $q \in \{0.25, 0.5, 1\}$. We use two laws: a uniform $U(-a, a)$ ($\gamma_4 = -1.2$) and a symmetric bimodal mixture (class C, $\gamma_4 \approx -1.62$). The held-out protocol is unchanged: each row uses 100 held-out replications with the Tikhonov ridge fixed by an internal validation split, training and test windows are distinct (no leakage), the Tikhonov ridge is fixed by an internal split, and the KUKP is reported against the fitted linear filter with 95% bootstrap CIs; since the evaluation is empirical, the carrier's non-Gaussianity requires no change to the derivation.

Table 2 and Fig. 4 confirm the synthetic finding on a real carrier: for both laws $e_{4,1} < 1$ (indicates an advantage over the fitted linear filter) at every q , and the 95% CI excludes unity. For both laws the mechanism beats the linear filter at every q , most strongly on the bimodal disturbance. The advantage is largest on the bimodal disturbance: $e_{4,1} = 0.853$ at $q = 0.5$, almost exactly reproducing the synthetic class C result (0.851), and widens to 0.725 at $q = 0.25$. The uniform disturbance, platykurtic in excess but unimodal, gives a consistently smaller, though still reliable, advantage (0.950 at $q = 0.5$). The result does not depend on a single file or channel ($e_{4,1} < 1$ on all six sources), the ridge never hits the grid ceiling, and the fitted linear filter matches the Wiener benchmark $e_1 = 1/(1 + q)$, so the KUKP is clean.

Table 2

Controlled experiment on a real carrier: the carrier is a CWRU normal baseline recording, the disturbance a controlled platykurtic law, logarithmic basis $s = 4$.

Disturbance law	q	$\bar{\gamma}_4$	$\text{cond}F^\varphi$	$e_{4,1}$ (95% CI)
bimodal	0.25	-1.62	8.3×10^5	0.725 (0.723, 0.728)
bimodal	0.50	-1.62	9.2×10^5	0.853 (0.852, 0.855)
bimodal	1.00	-1.62	1.0×10^6	0.949 (0.948, 0.950)
uniform	0.25	-1.20	6.5×10^5	0.926 (0.925, 0.927)
uniform	0.50	-1.20	7.8×10^5	0.950 (0.949, 0.951)
uniform	1.00	-1.20	9.3×10^5	0.979 (0.979, 0.980)

Discussion

Why compression matches symmetric noise. The compressive mechanism earns its place on a specific distributional geometry, and the odd symmetry of the elements is what delimits it. A sign-preserving odd family is orthogonal to the skew of the disturbance, so it can only exploit structure that survives reflection about the mean — which is why it improves on the linear filter on every symmetric class and stalls on the asymmetric one. Within the symmetric family the gain is largest where the bulk is informative, platykurtic and multimodal regime, $\gamma_4 < 0$ at low skew), the informative structure lives in the *shape of the bulk*, in particular in the spacing of resolvable modes, not in the tails. A slowly growing, sign-preserving nonlinearity $\text{sign}(\xi)\log^{i-1}(1 + |\xi|)$ is sensitive precisely there: it varies appreciably across the central region while flattening toward the extremes, so successive elements pick out gradations of the inter-mode structure that a smooth, broadly supported family averages out. This regime is singled out in [8]: under negative excess kurtosis the optimal L_p filter with $p \neq 2$ outperforms Wiener, and medley filters [9] and sequential filters for uniform noise [10] confirm this independently. This is the mechanism behind the class C result ($e_{4,1} = 0.851$ at $q = 0.5$, 0.729 at $q = 0.25$), and it is the reason we recommend the compressive basis for symmetric disturbances, and within them expect the largest margin on the platykurtic/multimodal corner. On asymmetric noise is regime-bound, not universal [11, 12].

Regularisation as a defining feature, not a patch. The body is severely ill-conditioned, $\text{cond}(F^\varphi) \sim 6 \times 10^4 - 9 \times 10^5$ at order $s = 4$, and this is the direct shadow of the property that makes the mechanism useful, not an incidental cost. The slow growth that lets $\log^{i-1}(1 + |\xi|)$ compress platykurtic structure is exactly what makes its successive integer powers nearly collinear over the support of a finite-variance ξ , so the covariance body crowds its small-eigenvalue end. The link between the condition number, the tails of the distribution, and the convergence rate of the Gram matrix to its population value is described in [13]; spectral frameworks [14] quantitatively relate the regulariser's eigenvalues to the stability of the solution. We therefore build the Tikhonov ridge (19) into the synthesis itself: the cross-validated λ , chosen following the methodology of [15, 16], damps the unstable directions, trading a controlled bias for a large reduction in the variance of $\hat{h}$, and it is what lets the regularised empirical e_s track the oracle (Table 1). The price is paid in convergence speed: the regularised curve approaches the theoretical value from above and more slowly than a well-conditioned mechanism would (Fig. 2), but never in the *existence* of the filter, which is guaranteed by finite variance alone. We therefore present the mechanism consistently as a regularised estimator throughout.

Finite-variance validity. Underneath the conditioning story is a structural guarantee the power basis does not enjoy. The body is finite under the single assumption of finite variance: because the elements grow more slowly than any power of ξ , the entries are dominated by $E\{\xi^2\} = \chi^2 + \pi^2$ and exist whenever the variance does, so no high-order moment is ever required to populate $F^\varphi h = R^\varphi$. Recent developments of moment-free Kunchenko stochastic polynomials via the empirical characteristic function [17] confirm that finite variance is the natural minimal assumption for generating-element filters. This cures the moment problem that limits the monomial basis, whose entry $\text{Cov}(\xi^i, \xi^j)$ calls on the moment of order $i + j$ and drives the body toward divergence on a Student- t or strongly leptokurtic law [11, 3]. The compressive family is consequently a valid generating mechanism on arbitrarily heavy tails, recovering, inside the Kunchenko stochastic-polynomial programme [4, 5, 18], the robustness that higher-order-statistics signal processing [3], M-estimators [2, 11, 19, 21], and L_p -filters [11] pursue by other means.

Negative excess is not a sufficient condition. The controlled nature of the disturbance in Section 4 matters, and the boundary deserves stating explicitly. We checked two sources of *raw* real noise with many platykurtic windows: the MIT-BIH NSTDB noise records (baseline wander, electrode motion, muscle artifact; up to 83% of windows with $\gamma_4 < 0$) and the CWRU baseline recordings themselves, used as the disturbance. In neither case does the logarithmic filter improve on the fitted linear filter: with a correctly tuned ridge, the KUKP stays at $\approx 1.00 - 1.03$. Negative excess kurtosis alone is therefore not a sufficient condition for an advantage. The compressive mechanism's benefit appears when the platykurtic or multimodal *shape* of the disturbance creates a nonlinear structure in the conditional mean that the linear filter does not capture (as in the mode-separated disturbance of class C), not whenever the excess kurtosis happens to be negative. This is why the real-data experiment is built as a controlled one: it tests the mechanism on a realistic *signal* shape without ascribing the required *noise* structure to any particular dataset.

Limitations and outlook. Four limitations bound the scope. First, the closed-form R^φ rests on the Gaussian-signal assumption $s \sim N(0, \chi^2)$; the body is distribution-free in the signal, but the cross-correlation would need re-derivation for a non-Gaussian s . Second, the filter is inertialess: it exploits the marginal non-Gaussianity of the noise but none of its temporal correlation, and an inertial extension coupling the compressive mechanism with a Wiener-type spectral stage [1] is the natural next step. Third, the verification is mostly synthetic: the controlled experiment on a real carrier (Section 4) confirms the mechanism on a realistic signal shape, but a fully uncontrolled real disturbance matched to the regime remains the most valuable future test, and the one we would prioritise. Fourth, the ridge λ is selected by held-out cross-validation; a data-driven joint selection of order and ridge that minimises e_s subject to a conditioning budget would make the mechanism fully self-tuning and is left open.

Conclusions

This paper has developed the polynomial inertialess filter $\hat{s} = h_0 + \sum_i h_i \varphi_i(\xi)$ for the compressive logarithmic generating mechanism of the Kunchenko space – the linear seed $\varphi_1 = \xi$ followed by the odd, monotone, sub-polynomial elements $\varphi_i = \text{sign}(\xi) \log^{i-1}(1 + |\xi|)$ – and benchmarked it against the Wiener linear filter through the KUKP $e_{s,1} = e_s/e_1$. The main results are as follows.

15. *Why the normal system exists at all.* The odd sub-polynomial elements $\text{sign}(\xi) \log^{i-1}(1 + |\xi|)$ keep the body $F^\varphi h = R^\varphi$ purely second-order, with the cross-vector following from the bounded derivative (18) via Stein's identity. Because every element grows slower than any power of ξ , each entry of F^φ and R^φ survives on finite variance alone (inequality (17)), so the filter is defined on arbitrarily heavy tails, including Student- t laws whose power-basis counterparts diverge; no high-order moment is ever invoked, which is what sets this family apart from the power basis.

16. *What that buys costs numerically.* The same slow growth that keeps the body finite also makes successive elements strongly collinear, so F^φ is severely ill-conditioned ($\text{cond}(F^\varphi) \sim 6 \times 10^4 - 9 \times 10^5$ at $s = 4$, growing with order). We stabilise the normal system with the cross-validated ridge (19) and treat it as part of the mechanism's design, not an add-on that patches over instability.

17. *Where it pays off, on synthetic and real carriers alike.* At $q = 0.5$, $s = 4$, the compressive filter beats the Wiener benchmark on the multimodal/platykurtic class C ($e_{4,1} = 0.851$) and on the heavy-tailed classes B (0.844) and D (0.962), and is only marginal on the asymmetric class A (0.992); the class C margin widens at low SNR, to $e_{s,1} = 0.729$ at $q = 0.25$. A controlled experiment on a real CWRU bearing carrier (Table 2, Fig. 4) reproduces the same regime ($e_{4,1} = 0.853$ on a bimodal disturbance, $q = 0.5$), whereas raw real noise with $\gamma_4 < 0$ (NSTDB, the CWRU baseline recordings themselves) gives no advantage: what matters is the disturbance's shape, not its excess kurtosis alone.

18. *And the theory holds up.* Wherever the regularised system is solved, the held-out empirical e_s matches the oracle within the bootstrap interval, approaching it from above (Table 1, Fig. 2); that slow, ridge-mediated approach is exactly what the conditioning in point 2 predicts.

Taken together, these results identify the logarithmic basis as a compressive specialist: a finite-variance-valid generating mechanism whose sub-polynomial shape is decisively well matched to symmetric non-Gaussian noise, and sharpest on its platykurtic and multimodal corner, at the cost of an explicit Tikhonov regularisation that the mechanism's own geometry makes unavoidable (Figs. 1, 2, 3).

What we take from this is not that the compressive basis should replace the Wiener filter generally, but that it is the right tool for a specific, identifiable regime. The natural next step is an adaptive switch that selects the

compressive mechanism from the estimated (β_3, γ_4) coordinates of the disturbance, deploying it precisely on the regime where its compression pays [4].

Data and code availability

The complete implementation of the compressive logarithmic generating mechanism, the Tikhonov-stabilised synthesis (19) and every experiment reported here is openly available at https://github.com/SZabolotnii/DSGE_PIF-code-supplement under the MIT licence. The repository provides the basis-agnostic solver of the normal system (9), the acceptance gate that verifies the order-1 reduction to the Wiener filter and the convergence of the empirical KUKP to its oracle value, one entry point per experiment, and the reference result files together with a verification script that compares a fresh run against them. All runs are seeded globally at 20260517 and are therefore deterministic and reproducible. The datasets are not redistributed: the CWRU bearing baseline recordings and the MIT-BIH NSTDB noise records are downloaded from their original sources on first run, and their identifiers and licences are documented in the repository.

ADDITIONAL INFORMATION

DECLARATION ON THE USE OF GENERATIVE ARTIFICIAL INTELLIGENCE TOOLS

In the preparation of this work, the authors used Claude for grammar and spelling checks, paraphrasing, and rephrasing of individual sentences. After using this service, the authors reviewed and edited the content and take full responsibility for the content of this publication

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КОМПРЕСИВНІ ЛОГАРИФМІЧНІ ПОРОДЖЕНІ ЕЛЕМЕНТИ ДЛЯ БЕЗІНЕРЦІЙНОЇ ФІЛЬТРАЦІЇ СИМЕТРИЧНИХ ЗАВАД

Розвивається поліноміальний безінерційний фільтр для відновлення гаусового сигналу з адитивного негаусового шуму в межах апарату розкладу Кунченка, причому за породжений елемент обрано компресивне, субполіноміальне сімейство: після обов'язкового лінійного елемента – непарні монотонні елементи. Оскільки ці елементи зростають повільніше за будь-який степінь, вони стискають, а не підсилюють хвости, і нормальна система синтезу скінченна тоді, коли шум має саму лише скінченну дисперсію – жоден момент понад другого ніколи не потрібен, тож тіло не може «розбігтися» на важких хвостах. Ціною цієї універсальності є чисельність: послідовні степені повільно змінної функції сильно колінеарні, тож тіло вкрай погано обумовлене й розв'язується під регуляризацією Тихонова з крос-валідованим гребенем. Ми показуємо, що ця контрольована ціна дає перевагу на режимі шуму, до якого компресивна форма узгоджена. Експерименти Монте-Карло з 95% бутстреп-довірчими інтервалами, зіставлені з лінійним фільтром Вінера через коефіцієнт Кунченка, показують, що логарифмічний базис є кращим фільтром на платикуртинному / мультимодальному шумі, де його повільно зростаюча нелінійність роздільноє міжмодову структуру, яка не відтворюється лінійним фільтром ($\rho = 0.851$ at $\rho = 0.5$, розширюючись до 0.729 – а 27% скорочення залишкової дисперсії над лінійним фільтром – за $q = 0.25$). Контрольований експеримент на реальному вузлі підшипника CWRU з платикуртинною завадою відтворює цей висновок ($\rho = 0.853$ на бімодальній заваді за $q = 0.5$), тоді як сирий реальний шум із самим лише від'ємним ексцесом переваги не дає – тобто виграш зумовлений структурою завади, а не ексцесом. Оскільки елементи непарні та знакозберігальні, механізм узгоджений із симетричними завадами як класом: він переважає лінійний фільтр на всіх трьох розглянутих симетричних класах і дає лише граничний виграш на асиметричному, де непарна компресія неефективно вловлює скіс. Тому ми рекомендуємо логарифмічний базис для симетричних негаусових завад, найсильніше – на їх платикуртинному та мультимодальному куті, і зазначаємо, що самого лише від'ємного ексцесу недостатньо для переваги.

Ключові слова: стохастичні поліноми; декомпозиція Кунченка; безінерційна нелінійна фільтрація; породжувальний елемент; логарифмічний базис; стискальна нелінійність; плосковершинний шум; мультимодальний шум; регуляризація Тихонова; симетричний шум; негаусівський шум; відношення сигнал/шум.